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GOCE/GRACE GGM evaluation over Attica and Thessaloniki, Greece and local geoid modelling in support of height unification

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Abstract: Within the frame of the “Elevation” project, supported by the action “Archimedes III – Funding of research groups in T.E.I.”, co-financed by the E.U. and Greek national funds, an extensive evaluation of the latest GOCE, GOCE/GRACE and combined Global Geopotential Models (GGM) has been carried out. The evaluation was performed using a set of collocated GPS and levelling BMs covering the regions of Attica and Thessaloniki. To this extent,the latest satellite-only and combined GOCE/GRACE GGMs were evaluated to conclude on the possible improvement brought by GOCE, given the two main methodologies used for the GGM development (DIR and TIM) and the latest releases of GOCE data (Release 5). For these GGMs, local height transformation parameters have been determined, employing low and higher order parametric models,in order to accommodate surveying and engineering applications. Moreover, local geoid models have been determined for the two study areas through the well-known Multiple-Input Multiple-Output System Theory (MIMOST) method, by utilizingGOCE GGMs and the local GPS/levelling data. The so-determined geoid models are validated against the set of available GPS/levelling BMs and conclusions are drawn with respect to the improvement brought by GOCE in resolving, with higher accuracy, the lower and medium band of the gravity field spectrum.

Keywords: Geopotential Models, Geoid Modelling, Height Unification, Thessaloniki, Attika

1. Introduction

The advent of Global Positioning System (GPS) technology provided a powerful tool forthe accurate estimation of height information. Nowadays, cm-level accuracymay be achieved using geodetic GPS receivers and differential measurement techniques even for baseline lengths of some hundreds of kilometers. The development of national continuously operating reference station networks forGlobal Navigation Satellite Systems (GNSS)measurements contribute to the stability and accuracy of the final solutions (Seeber, 2003; Fotiou and Pikridas, 2012). Nevertheless, the use of GNSS for estimating the vertical position can only provide pure geometric information, i.e., ellipsoidal (or geometric) heights. The ellipsoidal height refers to the distance between the measurement point and a mathematical model that approximates the Earth’s shape and therefore lacks a physical

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meaning which is needed in technical infrastructure works. In order to obtain a physically meaningful height, like for example the orthometric height, it is necessary to combine the ellipsoidal heights with geoid heights. Geoid heights (or undulations) refer to the geoid, an equipotential surfacethat approximates the mean sea surface and relatesthe geometrical tothe physical shape of the Earth. Gravity field modelling, a special discipline of Physical Geodesy, may be used for the accurate estimation of the geoid that will provide the connection ofthe GNSS measured heights to orthometric ones.

Different regional and local geoid solutions have been provided for the wider Hellenic areaduring the last two decades. Recently, the tremendous development of GNSS along with improved geoid solutions haveled to the determination of highly accurate orthometric heights. The geoid models used maybe derived nowadays from gravity data obtained from satellite gravity missions, e.g., the CHAMP, GRACE and GOCE satellites. The accuracies obtained cover a wide spectrum of geodetic applications and meet today's requirements for a large number of scientific works and projects related not only to geodesy, but also to other branches of the geosciences,such as oceanography, marine geophysics and geology, geodynamics, etc. The above mentioned orthometric height determination cannot replace, at least until now, the classical spirit levelling, since it is not possible yet to obtain first order accuracies. However, lower order accuracies, which are very satisfactory for a large number of geodetic and engineering applications, may be obtained through the combination of ellipsoidal heights from GNSS measurements and geoid heights computed from satellite derived geoid models.Therefore, these orthometric heights may replace the costly and time consuming geometric levelling. This is very important in areas with topographic peculiarities, where the ellipsoidal heights of GNSS can be determined with an accuracy close to some mm over distances of several km. Based on the same methodology as before, accurate relative heights can be computed by the same or slightly better accuracies, taking into account the accuracies of the local/regional geoid models used in the computations. The accuracies achieved forthe relative geometric heights, and consequently orthometric heights, can reach the level of 2-5 mm over distances of several km. In order to obtain relative geoid heights with the same accuracies as before, it is necessary to use a high degree and order geopotential model. The use of such a model, truncated at a specific degree, introduces errors in the computations related to the long-wavelengths of the gravity field spectrum. Additional errors may beintroduced in the computations by inconsistencies between the reference systems of gravity anomalies and GPS heights. In order to eliminate these types of errors, it is possible to compute parametric modelsthrough a least squares fitting procedure in order to minimize the differences between the gravimetric geoid heights andthe corresponding GPS/levelling heights.

With the GOCE mission having reached its end in late 2013, the unprecedented contribution of the first mission to carry-on gradiometric observations in space was and is still being evaluated. GOCE contributed significantly not only in the field of geodesy, where its impact on gravity field and geoid modelling was long expected, but to oceanography, geophysics and even time-variable gravity field modelling. Its contribution to geodesy has been predominant, since GOCE provided improved representations of the Earth's gravity field especially in the long-to-medium and medium wavelengths of the spectrum. In the pure geodetic context, the contribution of GOCE is viewed in the improved representation of the Earth's gravity field functionals and especially gravity anomalies and geoid heights. These improvements are commonly assessedby examining differences obtained with external validation datasets, such as geoid heights derived from GPS/levellingmeasurements (Andritsanos et al., 2014; Tziavos et al., 2015; Vergos et al., 2014) on trigonometric benchmarks (BMs), or with the aidofa very high-degree Global Geopotential Model (GGM) from the pre-GOCE era, i.e., EGM2008 (Pavlis et al., 2012). The latter model is very useful for such GOCE GGM validation experiments since it offers an independent source of information that is not included in the development of the GOCE-derived GGMs. Moreover, GOCE data are now commonly used for the determination of the zero-level geopotential value towards the unification of Local Vertical Datums (LVD) to a global one (Vergos et al. 2014; Grigoriadis et al., 2014; Gruber et al., 2012; Hayden et al., 2012; Tocho and Vergos, in press).

The first part of the study is devoted to the evaluation of the 5th release of GOCE GGMs and the combined EIGEN-6C4 model. A combination procedure with the signal of EGM08 is used in order to consider the improvement in the low frequency part of the gravity signal spectrum caused by the incorporation of GOCE data. The second part is devoted to the determination of a combined GGM/GPS/levelling geoid using Multiple Input Multiple Output Systems Theory (MIMOST) (Sideris, 1996; Andritsanos and Tziavos, 2000). The final combined geoid solution may be used directly as it contains information from GGM and GPS/leveling.

2. Available data and Models

2.1. GPS/levelling data

In the present study, 230 geoid height values from GPS/levellingmeasurements were used out of which 103 are within the area of Attica and the remaining 127 are within the area of Thessaloniki. The point values refer to benchmarks of the national trigonometric network of Greece and their distribution for the two study areas, Attica and Thessaloniki, is shown in Figure 1. Thevalues for the orthometric heights are the ones provided by the Hellenic Military Geographic Service, while the values for the ellipsoidal heights (GPS data) were measured during the “Elevation” project (Anastasiou et al., 2013). This is a completely independent set of GPS/Levelling observations than the HEPOS-based GPS/Levelling database used during the latest GGM evaluation over Greece (see e.g., Tziavos et al., in press; Vergos et al., 2014) and therefore the results acquired will provide a new independent look on the GOCE GGM performance. The main difference of this new dataset is that longer GPS observations (larger than two hours compared to one hour) have been carried out, while spirit levelling campaigns between the BMs, wherever possible due to the distance limitations, were performed to validate the available orthometric heights.These GPS/leveling data have already been used for the evaluation of the complete set of GOCE, GOCE/GRACE and combined GGMs in Vergos et al. (2015), where the superior performance of the latest DIR-R5 and TIM-R5 GGMs has been shown (see Figure 2, ibid.).

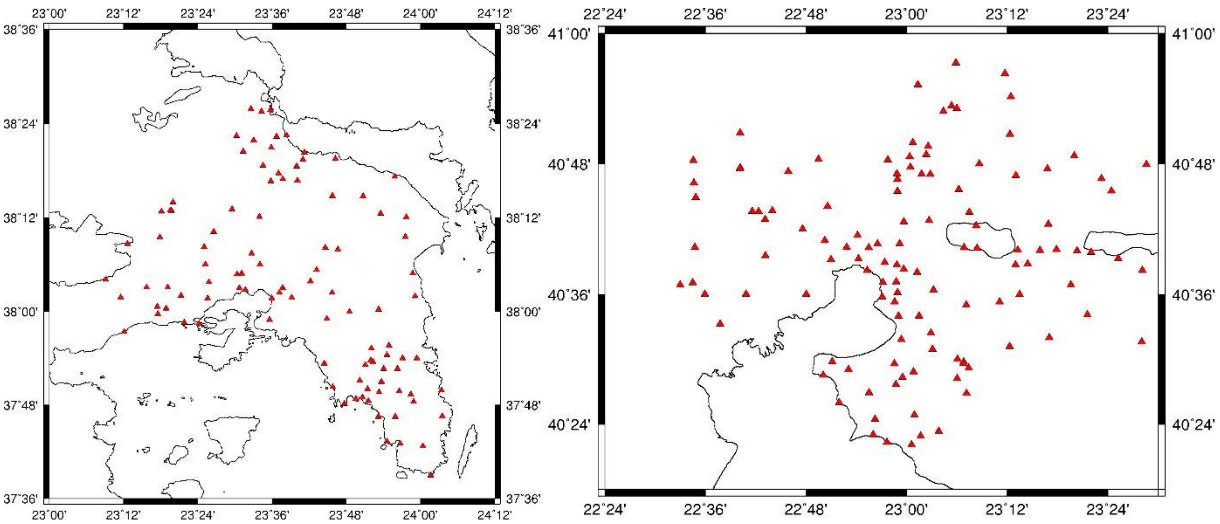


Figure 1. Distribution of the available geoid height values from GPS/levelling data for the two study areas (left: Attica, right: Thessaloniki).

2.2. Geoid heights from Geopotential Models

For the computation of geoid heights, the latest, till today, TIM and DIR model releases were used along with EGM08 and EIGEN6C4. Information pertaining to their maximum spherical harmonic degree of expansion and the source of data used for deriving them are provided in Table 1. Following a spectral analysis and the evaluation results given by Vergos et al. (2014) and Tziavos et al. (2015), two more models were also used in the computations. These two models were produced after combining the TIM (up to degree 140) with EIGEN08 and DIR (up to degree 140) with EIGEN08 as well.The derivation of geoid undulations (N_i) were computedin a tide-free system using the *harm_synth* software (Pavlis et al., 2012), according to the following formula (Heiskanen and Moritz 1967, Eqs. 8.100):

$$N_i = \zeta_i + \frac{\Delta g_B}{\bar{\gamma}} H_i, \tag{1}$$

where, for the ith benchmark, Δg_B is the Bouguer gravity anomaly, H_i is the orthometric height, $\bar{\gamma}$ is the mean normal gravity along the plumb line between the ellipsoid and the telluroid and ζ_i is the height anomaly computed as follows:

$$\zeta_i = \frac{GM_0}{\gamma r} \sum_{m=0}^{n_{max}} \left(\frac{\alpha}{r}\right)^n \sum_{m=0}^{n} (\overline{\Delta C}_{nm} \cos m\lambda + \overline{\Delta S}_{nm} \sin m\lambda) \bar{P}_{nm} (\cos\theta). \tag{2}$$

In the previous equation, n_{max} is the maximum degree of expansion of the GGM,GM_0 is the geocentric gravitational constant, γ is the normal gravity, $\overline{\Delta C}_{nm}$ and $\overline{\Delta S}_{nm}$ are the differences between the fully normalized potential coefficients and those implied by the reference equipotential ellipsoid, $\bar{P}_{nm}$ are the fully normalized associated Legendre functions, α is a scaling factor associated with $\overline{\Delta C}_{nm}$ and $\overline{\Delta S}_{nm}$, and r, θ, λ are the polar coordinates of the benchmark.

Table 1. GGMs used for evaluation.

Model	n_{max}	Data	Reference
DIR-R5	300	S(GOCE, GRACE, LAGEOS)	Bruinsma et al, 2013
TIM-R5	280	S(GOCE)	Brockmann et al., 2014
EGM08	2190	S(GRACE), G, A	Pavlis et al., 2012
EIGEN-6C4	1420	S(GOCE, GRACE, LAGEOS), G, A	Förste et al, 2014

Data: S = Satellite Tracking Data, G = Gravity Data, A = Altimetry Data

GRACE (Gravity Recovery And Climate Experiment), CHAMP (CHAllenging Mini-satellite Payload), GOCE (Gravity field and steady state Ocean Circulation Explorer), LAGEOS (Laser GEOdynamics Satellite)

3. Evaluation of geoid heights derived from Global Geopotential Models

Based on the available GPS/leveling data, an external validation of selected GGMs was carried out for the two study areas. The validation involved the assessment of both absolute and relative differences between the geoid heights derived from the GGMs and those from GPS/leveling by applying also a least-squares fitting procedure for adjusting the computed absolute differences to various parametric models. A detailed description of the methodology followed is provided next.

3.1. Methodology

The first step in the evaluation of geoid heights derived from GGMs (N_i^{GGM}) was the computation of their differences with those provided from GPS/leveling measurements ($N_i^{GPS/lev}$). The absolute differences for each benchmark (ΔN_i^{abs}) were computed according to the following equation

$$\Delta N_i^{abs} = N_i^{GPS/lev} - N_i^{GGM} - N_i^0, \qquad (3)$$

where N_i^0 is the contribution of the zero-degree harmonic to the GGM geoid undulations with respect to a specific reference ellipsoid. N_i^0 may be computed using the following formula

$$N_i^0 = \frac{GM - GM_0}{R_E \gamma_i} - \frac{W_0 - U_0}{\gamma_i}, \qquad (2)$$

where γ_i is the normal gravity computed on the reference ellipsoid according to the closed formula of Somigliana and R_E is the mean radius of the Earth. The term GM corresponds to the geocentric gravitational constant of the geoid while GM_0 to that of the reference ellipsoid. W_0 and $U_0$are the gravity potential of the geoid and ellipsoid of reference, respectively.

In the next step, different parametric models were used for determining corrector surfaces for the computed differences ΔN_i^{abs}. The determinationwas carried out following a least-squares fitting procedure for estimating the coefficients of the parametric models. The corresponding system of equations may be written in matrix notation as

$$\boldsymbol{L} = \boldsymbol{A}^T \boldsymbol{x} + \boldsymbol{v}, \tag{5}$$

where $\boldsymbol{L}$ is the vector of observations containing the differences ΔN_i^{abs}, $\boldsymbol{A}$ is the design matrix, $\boldsymbol{x}$ the vector of unknown coefficients of the parametric model and $\boldsymbol{v}$ the vector of the residual errors. A total of six parametric models were used in the computation for each area. These models are: (A)the four-parameter similarity transformation model, (B)the five-parameter similarity transformation model, (C)a model dependent on orthometric and geoid heights, (D) a model dependent only on orthometric heights, (E) a model dependent only on geoid heightsand (F)athird order polynomial model. The parametricmodelsare presented by the following equations (Fotopoulos, 2003; Kotsakis and Katsambalos, 2010):

$$A{:}\ \boldsymbol{\alpha}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{x} = x_0 + x_1 \cos \varphi_i \cos \lambda_i + x_2 \cos \varphi_i \sin \lambda_i + x_3 \sin \varphi_i \tag{6}$$

$$B{:}\ \boldsymbol{\alpha}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{x} = x_0 + x_1 \cos \varphi_i \cos \lambda_i + x_2 \cos \varphi_i \sin \lambda_i + x_3 \sin \varphi_i + x_4 \sin^2 \varphi_i \tag{7}$$

$$C{:}\ \boldsymbol{\alpha}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{x} = \mu + \delta s_H H_i + \delta s_N N_i \tag{8}$$

$$D{:}\ \boldsymbol{\alpha}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{x} = \mu + \delta s_H H_i \tag{9}$$

$$E{:}\ \boldsymbol{\alpha}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{x} = \mu + \delta s_N N_i \tag{10}$$

$$F{:}\ \boldsymbol{\alpha}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{x} = \sum_{m=0}^{2} \sum_{n=0}^{2} x_{(3m+n)} (\varphi_i - \varphi_0)^n (\lambda_i - \lambda_0)^m \cos^m \varphi_i \tag{11}$$

wherex_l $\{l$:0,…,8}, δs_H and δs_N are the model coefficients, φ_i and $\lambda_i$denote the geographical latitude and longitude, respectively, and φ_0 and$\lambda_0$are the corresponding mean geodetic latitude and longitude of the study area.

The last step involved the computation of the relative geoid height differences, between the GGM geoid models and the heights obtained from GPS/levelling, for different pairs of benchmarks (i, j) based on the following equation

$$\Delta N_{ij} = \left(N_j^{GGM} - N_i^{GGM}\right) - \left(N_j^{GPS/lev} - N_i^{GPS/lev}\right). \tag{12}$$

These differences were then used in the computation of the relative accuracy by the following equation:

$$a_r = \frac{\Delta N_{ij}}{S_{ij}} [ppm], \quad (13)$$

where S_{ij} is the spherical distance between benchmarks i and j. The spherical distancemay be computed using the formula

$$S_{ij} = R_E \operatorname{acos}(\sin\varphi_i \sin\varphi_j + \cos\varphi_i \cos\varphi_j \cos(\lambda_i - \lambda_j)). \quad (14)$$

3.2. Results and discussion

Following the methodology presented in the previous section, the absolute differences between the geoid heights and the GPS/levelling benchmarks were computed for each test area. The statistical values of the differences (before fit) are provided in Table 2. From the results shown, it is observed that the differences in Thessalonikiare larger than the ones observed in Atticain terms of standard deviation, mean value and range. A possible explanation for these differencesis directly related to the vertical network of Greece.The benchmarks for the area of Attica are close to the reference point of the Greek vertical datum, i.e., the tide gauge station at Piraeus port. On the other hand, the benchmarks located in the area of Thessaloniki lie approximately more than 300 km away from the reference point anda common adjustment of the Greek vertical network has never been carried out so far (Tziavos et al., 2012). Therefore, the differences found may be first attributed to inconsistencies of the Greek vertical datum while it is also possible that the models do not represent sufficiently the gravity field in the area of Thessaloniki.

Table 2. Statistics of geoid height differences between GPS/levelling geoid heights and GGMs geoid heights before and after the least-squares fit with parametric models for the areas of Attica and Thessaloniki.

		Attica				**Thessaloniki**			
Geoid Model	**Parametric Model**	**mean [m]**	**std [m]**	**range [m]**	R_{adj}^2	**mean [m]**	**std [m]**	**range [m]**	R_{adj}^2
DIR-R5 max deg: 140	Before Fit	-0.575	0.334	1.476	-	-0.765	0.447	2.241	-
	Model A	0.008	0.177	0.819	0.727	0.066	0.243	1.354	0.691
	Model B	0.000	0.173	0.823	0.744	0.000	0.239	1.338	0.723
	Model C	0.000	0.250	1.097	0.452	0.000	0.314	1.616	0.513
	Model D	0.000	0.333	1.437	0.018	0.000	0.331	1.746	0.456
	Model E	0.000	0.286	1.388	0.278	0.000	0.445	2.201	0.017
	Model F	0.000	0.122	0.524	0.878	0.000	0.180	1.088	0.849

Table 2. (continued)

Geoid Model	Par. Model	mean	std	range	R_{adj}^2	mean	std	range	R_{adj}^2
DIR-R5 max deg: 300	Before Fit	-0.262	0.187	0.887	-	-0.528	0.472	2.396	-
	Model A	0.003	0.160	0.803	0.284	0.045	0.259	1.405	0.697
	Model B	0.000	0.160	0.801	0.292	0.000	0.259	1.399	0.710
	Model C	0.000	0.171	0.766	0.175	0.000	0.339	1.764	0.494
	Model D	0.000	0.184	0.840	0.044	0.000	0.364	1.999	0.411
	Model E	0.000	0.181	0.774	0.068	0.000	0.466	2.303	0.032
	Model F	0.000	0.115	0.538	0.648	0.000	0.180	1.088	0.864
DIR-R5 max deg: 140 + EGM08	Before Fit	-0.406	0.080	0.514	-	-0.488	0.160	1.014	-
	Model A	0.005	0.080	0.516	0.027	0.042	0.153	0.772	0.044
	Model B	0.000	0.079	0.507	0.065	0.000	0.153	0.774	0.120
	Model C	0.000	0.078	0.508	0.072	0.000	0.138	0.719	0.269
	Model D	0.000	0.078	0.517	0.056	0.000	0.138	0.731	0.260
	Model E	0.000	0.080	0.514	0.010	0.000	0.151	0.723	0.116
	Model F	0.000	0.072	0.397	0.255	0.000	0.141	0.738	0.269
TIM-R5 max deg: 140	Before Fit	-0.563	0.336	1.535	-	-0.768	0.449	2.250	-
	Model A	0.007	0.177	0.819	0.729	0.066	0.243	1.354	0.694
	Model B	0.000	0.173	0.823	0.746	0.000	0.239	1.338	0.726
	Model C	0.000	0.250	1.097	0.456	0.000	0.316	1.622	0.514
	Model D	0.000	0.335	1.446	0.016	0.000	0.333	1.757	0.455
	Model E	0.000	0.285	1.383	0.286	0.000	0.447	2.205	0.018
	Model F	0.000	0.122	0.524	0.879	0.000	0.180	1.088	0.850
TIM-R5 max deg: 280	Before Fit	-0.293	0.202	0.989	-	-0.671	0.446	2.253	-
	Model A	0.004	0.160	0.805	0.394	0.058	0.258	1.404	0.656
	Model B	0.000	0.159	0.796	0.407	0.000	0.258	1.403	0.675
	Model C	0.000	0.182	0.805	0.205	0.000	0.322	1.753	0.486
	Model D	0.000	0.200	0.903	0.033	0.000	0.333	1.891	0.447
	Model E	0.000	0.192	0.818	0.103	0.000	0.445	2.228	0.010
	Model F	0.000	0.116	0.543	0.698	0.000	0.180	1.095	0.847
TIM-R5 max deg: 140 + EGM08	Before Fit	-0.394	0.080	0.776	-	-0.491	0.160	0.731	-
	Model A	0.005	0.080	0.517	0.029	0.042	0.153	0.771	0.047
	Model B	0.000	0.079	0.508	0.067	0.000	0.153	0.774	0.124
	Model C	0.000	0.079	0.512	0.065	0.000	0.138	0.715	0.275
	Model D	0.000	0.079	0.524	0.044	0.000	0.138	0.729	0.266
	Model E	0.000	0.080	0.517	0.011	0.000	0.151	0.718	0.118
	Model F	0.000	0.072	0.397	0.258	0.000	0.141	0.738	0.272
EIGEN6C4	Before Fit	-0.317	0.083	0.548	-	-0.513	0.153	0.730	-
	Model A	0.004	0.080	0.522	0.095	0.044	0.149	0.774	0.005
	Model B	0.000	0.079	0.512	0.135	0.000	0.149	0.774	0.084
	Model C	0.000	0.080	0.515	0.110	0.000	0.138	0.749	0.201
	Model D	0.000	0.083	0.549	0.010	0.000	0.138	0.744	0.193
	Model E	0.000	0.080	0.517	0.087	0.000	0.147	0.755	0.090
	Model F	0.000	0.072	0.400	0.324	0.000	0.138	0.740	0.247

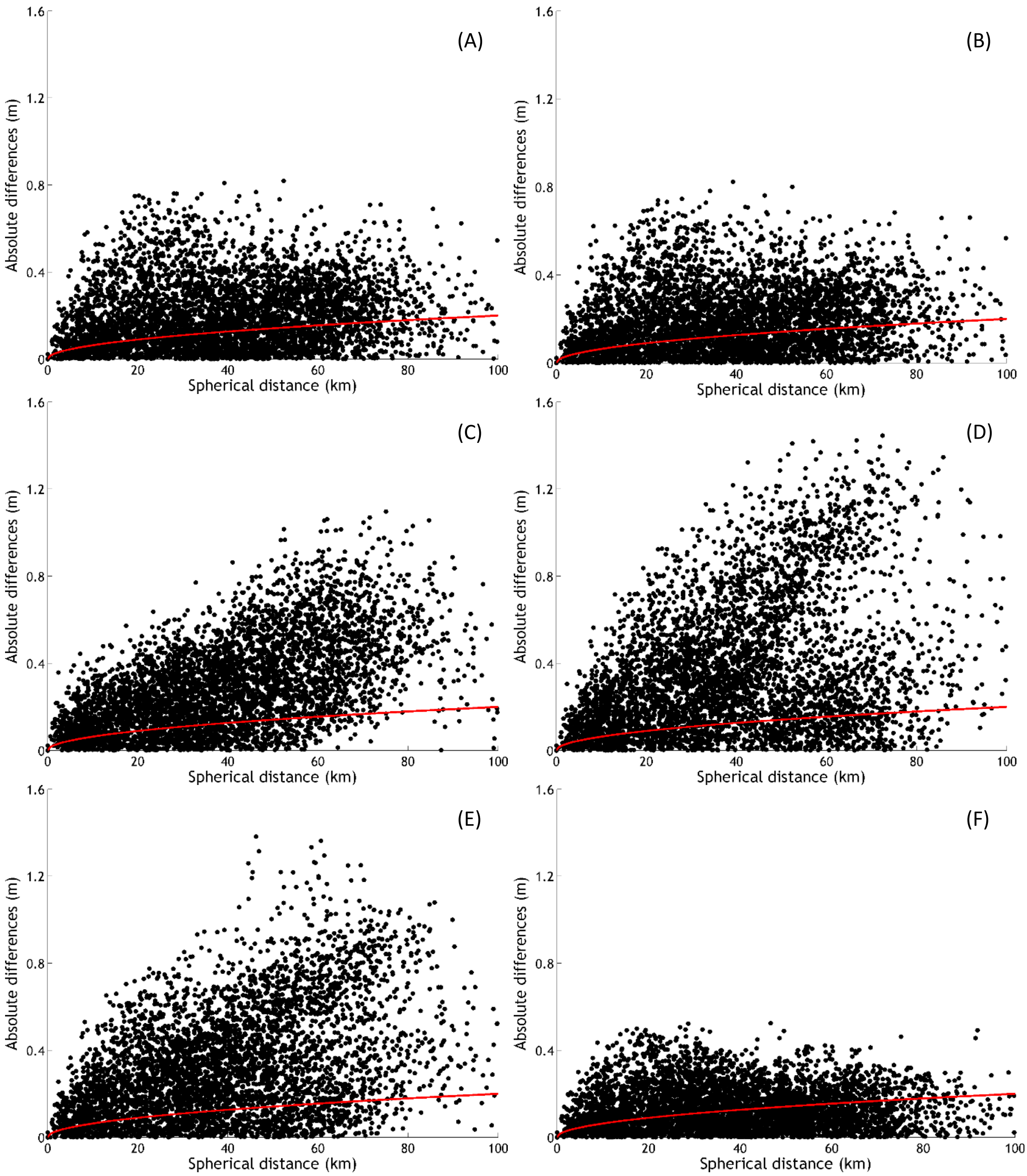


Figure 2. Absolute differences between geoid heights from the TIM-R5 (max degree 140) geoid model and the GPS/levelling geoid heights in the area of Attica after the least-squares fit to different parametric models. The red line represents the standard error while the type of the model used is noted on the upper right corner of each diagram.

A least-squares adjustment procedure was then carried out using the six parametric models described in the previous section for the geoid models examined in our study. The statistical results of the absolute differences after the fitting procedure are given in Table 2, while a sample visual representation of the differences is illustrated in Figure 2 for the area of Attica and for the TIM-R5 (max degree 140) geoid model. By examining the statistical results, all the parametric models present a consistent behavior for the two study areas apart from models D and E. Although the performance of models D and E is not adequate for both areas, considering the adjusted coefficient of determinationR_{adj}^2, they reveal a difference on the dependence to the geoid and orthometric heights for the two areas. As previously described, model D is an orthometric height dependent model while model E is a geoid height dependent model. The better performance of model E for the area of Thessaloniki designates that the orthometric heights, for this area, have errors that may be partly absorbed by the parametric model. On the contrary, model D is unable to reduceany discrepancies that are based on geoid heights. For Attica, the results present an opposite behavior. Model D is unable to improve further the results while model E leads to a small improvement. This remark supports the previously given conclusion that the large differences between the statistics for the two areas may be attributed to problems in the Greek vertical network.

By further examining the results of Table 2, the third order polynomial model (model F) seems to provide the best fitting results for both test areas (see for example Figure 2).In the case where the two GOCE-based models DIR-R5 and TIM-R5 are combined with EGM08, the parametric models provide an improvement to the results of approximately 1 cm for Attika in terms of standard deviation and 2 cm for Thessaloniki. In Figure 3indicative plots are shown for the estimated corrector surfaces for the area of Attica using a third order polynomial parametric model (model F). The corrector surfaces for model F depict a south-west to north-east trend, while for the area of Thessaloniki no such trend is detected.

In Figure 4 the spatial distribution of the differences between geoid heights from the combined TIM-R5 model (max degree 140) and EGM08 and the GPS/levelling data is presented. The differences shown are the ones obtained after the fit to a third order polynomial model (Model F). For the area of Attica, the fit to Model F removes the previously described south-west to north-east trend, while for both areas no spatial pattern for the differences is detected. The absence of a spatial pattern for the differences observed in the area of Thessaloniki along with the previous results leads to the conclusion that it is necessary to further investigate the differences by re-measuring parts of the Greek vertical network.

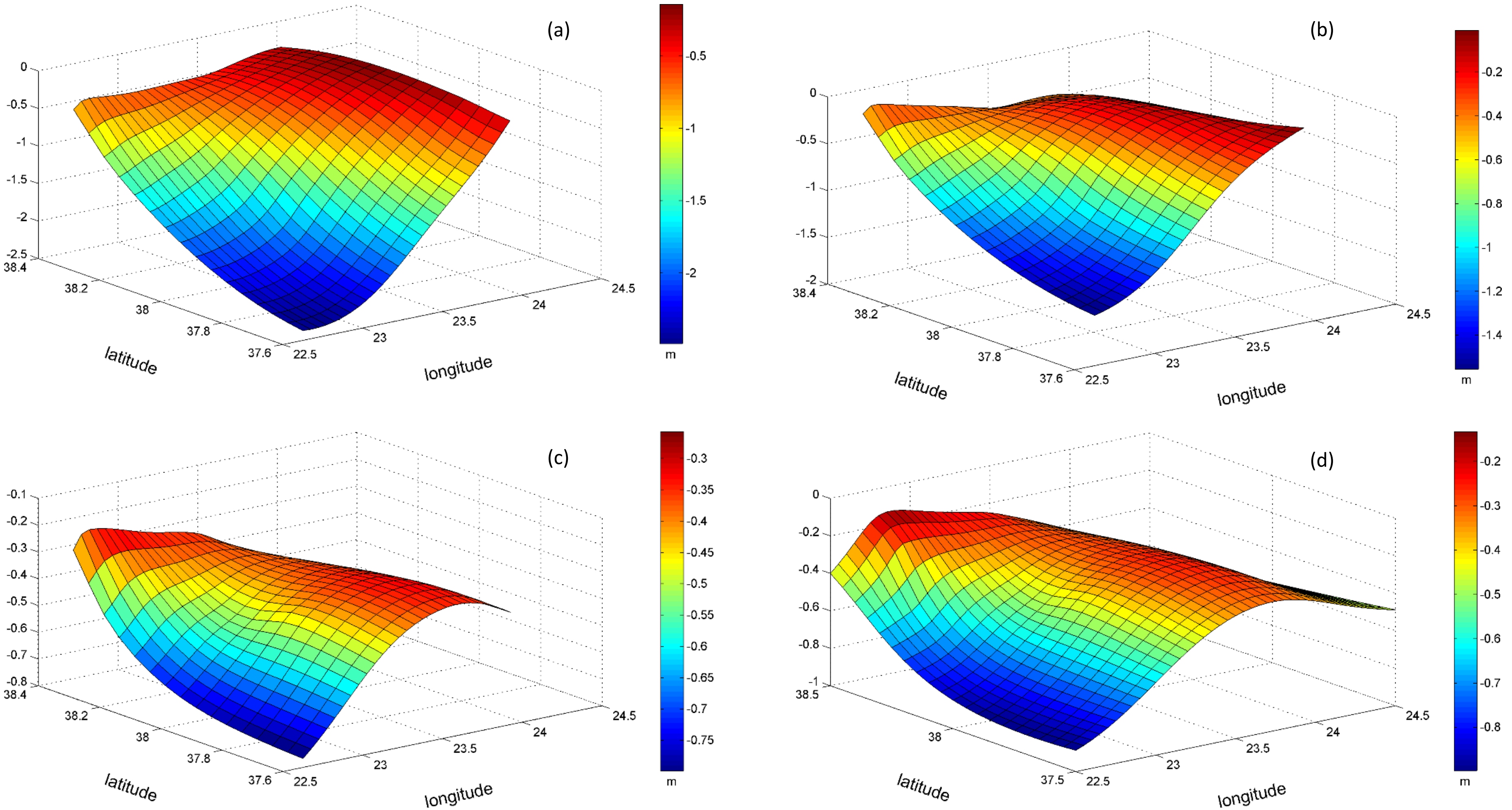


Figure 3. Corrector surface computedfor the area of Atticausing a third order polynomial parametric model (model F) forthe differences between geoid heights from GPS/levelling and the geoid models: a) TIM-R5 (max degree 140), b) TIM-R5 (max degree 280), c) combination of TIM-R5 (max degree 140) and EGM08 and d) EIGEN6C4.

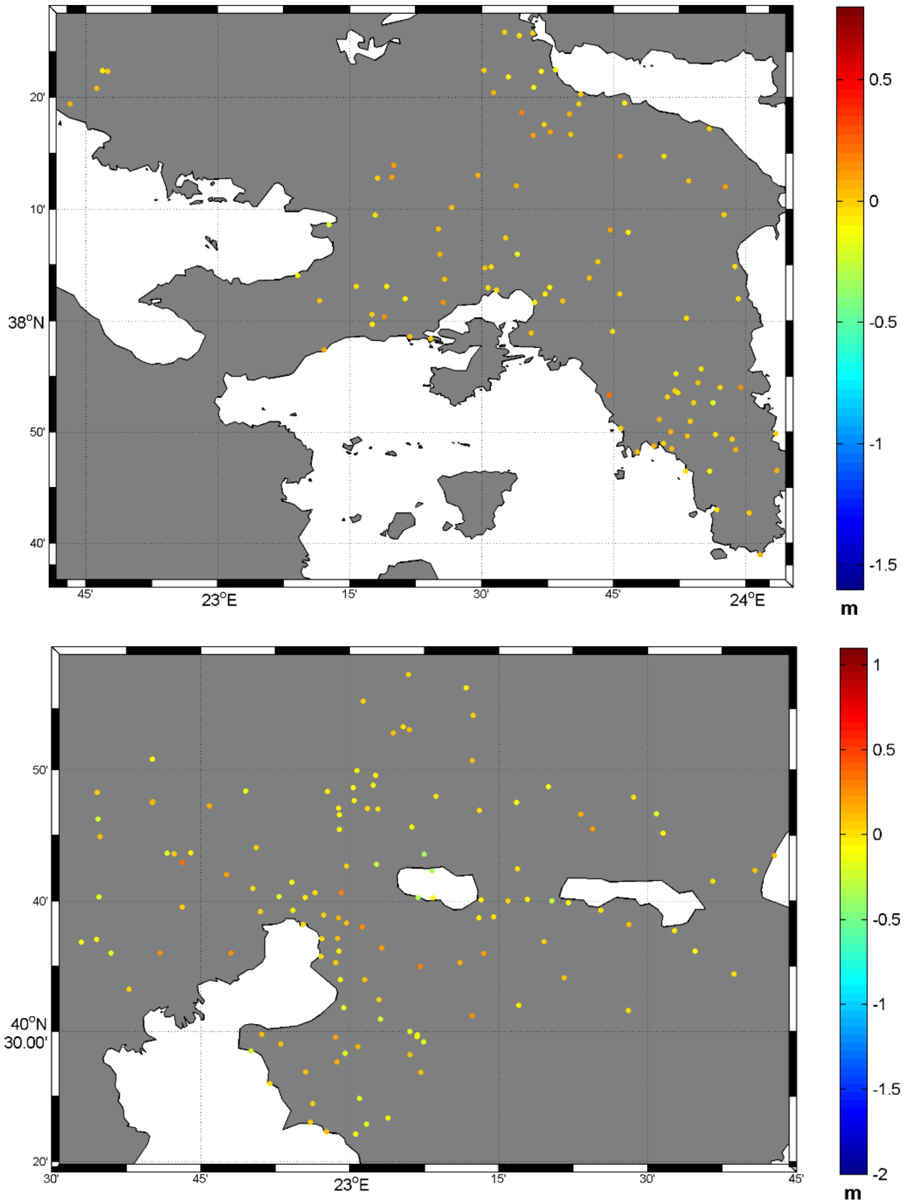


Figure 4. Spatial distribution of the absolute differences between geoid heights from the combined TIM-R5 (max degree 140) and EGM08 geoid model and the GPS/levelling geoid heights after the least-squares fit to the third order polynomial parametric model (model F) for Attika (top) and Thessaloniki (bottom).

Next, the relative accuracy was estimated for the relative differences computed after the least-squares fit to the parametric models. The relative accuracy for the area of Thessaloniki varies between 12 and 16 ppm for baselines 10 to 20 km long, while for the area of Attica the corresponding range is from 9 to 6 ppm. An indicative plot of the relative accuracy with respect to the baseline length is provided in Figure 5for the benchmarks in Atticaand for the TIM-R5 model (maximum degree 140 and 280), its combination with EGM08 and EIGEN6C4 after a third-order polynomial fit. In the two test areas the GOCE-based models present a rapid decrease in the relative accuracy for baselines shorter than 60 to 70 km long with respect to the combination model (GOCE-based and EGM08) and

EIGEN6C4. This may be attributed to the lack of surface gravity data in the GOCE-based models. When comparing though the full spectrum GOCE-based model with the one up to degree 140, we observe only a slight improvement of 1 ppm for baselines shorter than 30 km. On the other hand, the relative accuracy of the combination model and of EIGEN6C4 present a rapid decrease for baselines of less than 30 km length. Similarly, the same results for the relative accuracy are obtained for both DIR-R5 and its combination with EGM08.

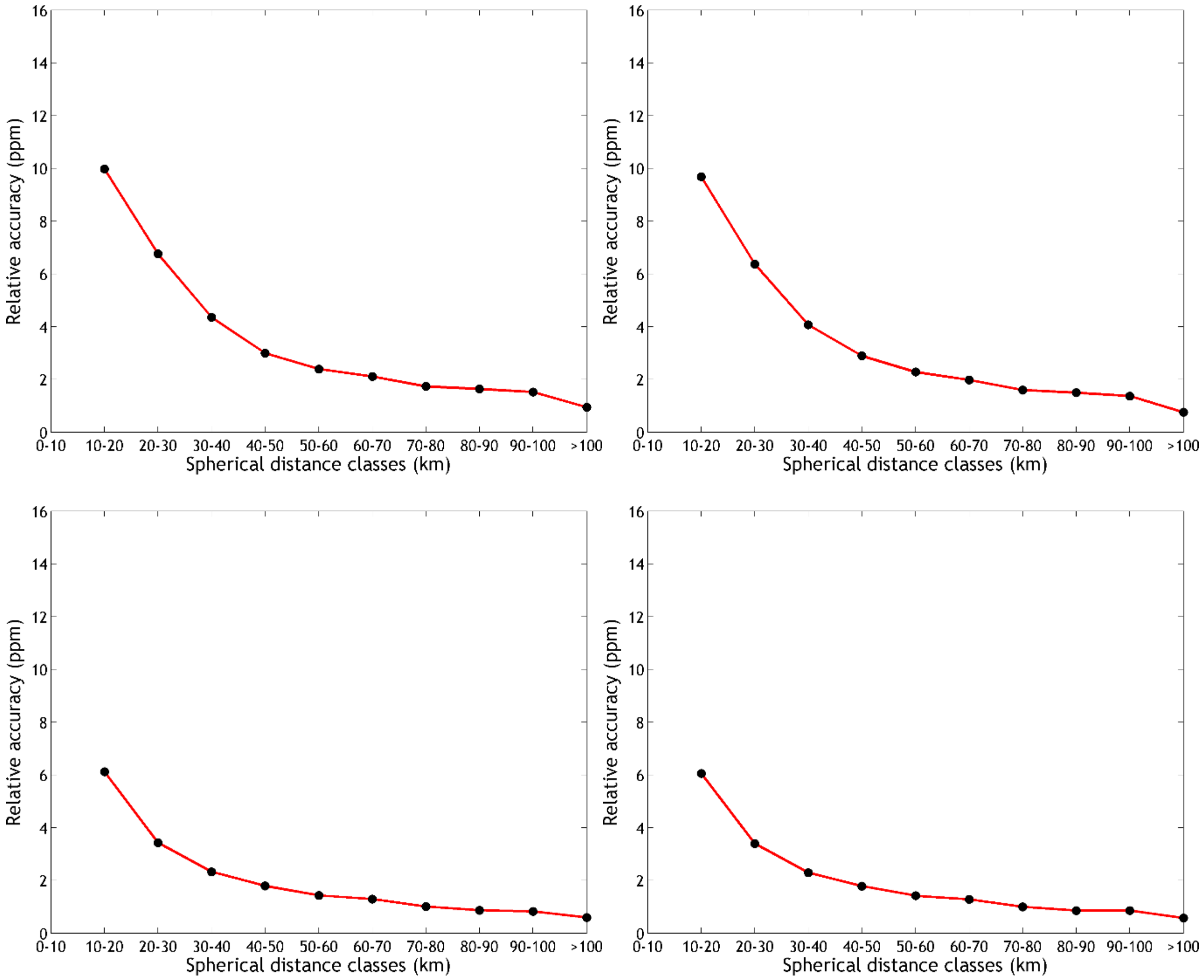


Figure 5. Relativeaccuracywith respect to baseline lengthfor the area of Attika after the least-squares fitting using a third order polynomial parametric model (model F) for the differences corresponding to the geoid models: a) TIM-R5 (max degree 140), b) TIM-R5 (max degree 280), c) combination of TIM-R5 (max degree 140) with EGM08and d) EIGEN6C4.

4. Input-Output geoid models

The Input-Output System Theory (IOST) method is primarily based on the spectral combination of heterogeneous data taking into account their statistical properties and approximating the Power Spectral Density (PSD) functions of the signals and their errors. These methods can handle heterogeneous data given on the same grid and propagate data errors into the results (see, Bendat and Piersol, 1986; Sideris, 1996). The systems theory solution is based on the assumption that both the input signals and their errors are stochastic variables. Additionally, PSD functions of both signals and their errors are known and the solution depends on the ratio of the error PSD and the signal PSD. In the case that the inputs are correlated, the systems theory solution is formally equivalent to the classical Least Squares Collocation (LSC) approach or to its stepwise alternative (Sideris, 1996; Sansò and Sideris 1997). The generalization of the IOST to the Multiple Input- Multiple Output System Theory (MIMOST) for geodetically oriented applications was proposed by Andritsanos and Tziavos (2000). Extensive derivations of the output signals and their accuracy estimations both with numerical tests on the determination of gravity field observables in land and sea areas are given by Andritsanos et al. (2000, 2001).

In this study a Multiple Input-Multiple Output System (MIMOS) is proposed, where the input measurements as well as the input and output signals are different geoid observables. More specifically, as is depicted in Figure 6, the input geoid signals contain information derived from the best GGMs combination of section 3.2., i.e. the contribution of GOCE DIR (r5) model until degree 140 and the residual signal of EGM08, in addition to geoid heights derived from GPS/levelling.

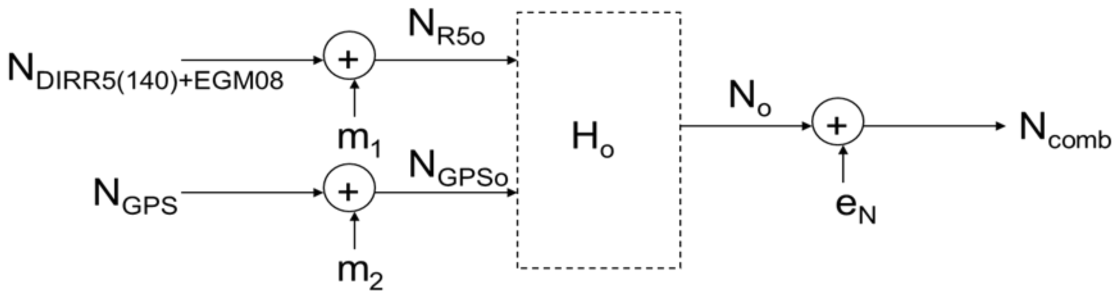


Figure 6. Schematic representation of the input – output system used in the computations of the combined geoid.

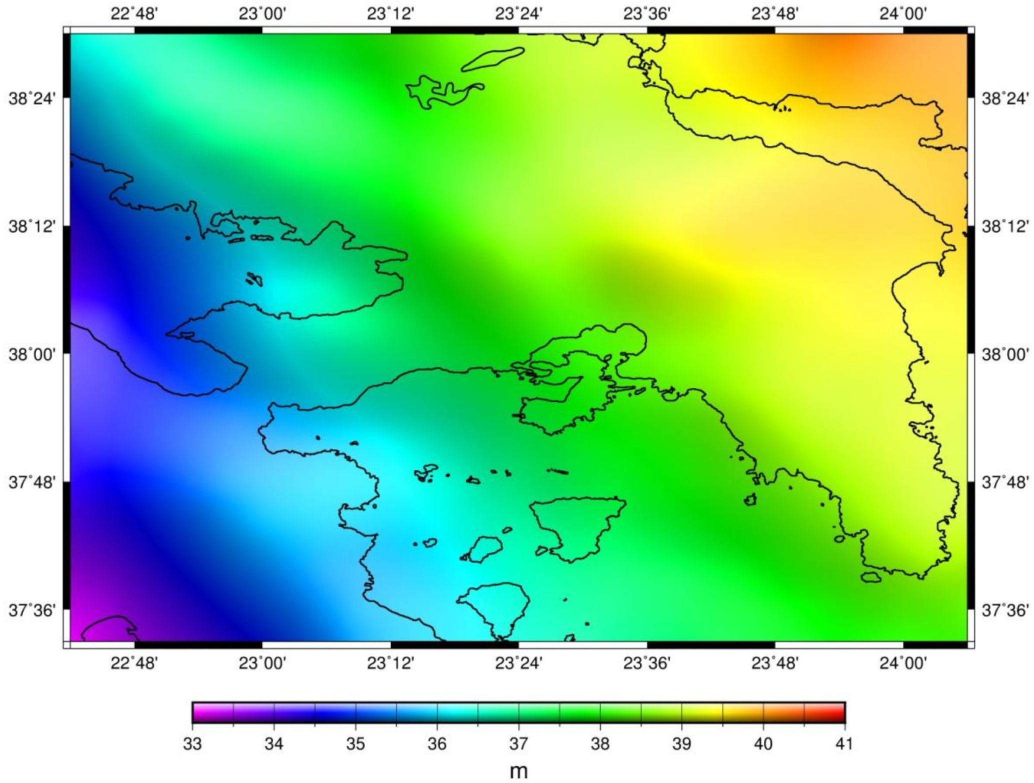
The optimal transfer function of the system$\widehat{\mathbf{H}}_{\mathbf{0}}$is computed by the application of a minimization criterion of the output error PSD. The optimal transfer function of the specific MIMOST is given in matrix form:

$$\widehat{\mathbf{H}}_{\mathbf{0}} = \mathbf{P}_{\mathbf{xy}}\left(\mathbf{P}_{\mathbf{yy}} + \mathbf{P}_{\mathbf{mm}}\right)^{-1} = \mathbf{P}_{\mathbf{xy}}\mathbf{P}_{\mathbf{y_0y_0}}^{-1}. \quad (13)$$

The final combined solution and the estimation of the error PSD and 2D covariance function is:

$$\widehat{\mathbf{X}}_{\mathbf{0}} = \widehat{\mathbf{H}}_{\mathbf{0}}\mathbf{Y}_{\mathbf{0}} = \mathbf{H}_{\mathbf{xy}}\left(\mathbf{P}_{\mathbf{y_0y_0}} - \mathbf{P}_{\mathbf{mm}}\right)\mathbf{P}_{\mathbf{y_0y_0}}^{-1}\mathbf{Y}_{\mathbf{0}}$$
$$\mathbf{P}_{\hat{\mathbf{e}}\hat{\mathbf{e}}} = \left[\mathbf{H}_{\mathbf{xy}}\left(\mathbf{P}_{\mathbf{y_0y_0}} - \mathbf{P}_{\mathbf{mm}}\right) - \widehat{\mathbf{H}}_{\mathbf{0}}\mathbf{P}_{\mathbf{y_0y_0}}\right]\left(\mathbf{H}_{\mathbf{xy}}^{*\mathbf{T}} - \widehat{\mathbf{H}}_{\mathbf{0}}^{*\mathbf{T}}\right) + \widehat{\mathbf{H}}_{\mathbf{0}}\mathbf{P}_{\mathbf{mm}}\mathbf{H}_{\mathbf{xy}}^{*\mathbf{T}} \quad (14)$$
$$\mathbf{C}_{\hat{\mathbf{e}}\hat{\mathbf{e}}} = \mathbf{F}^{-\mathbf{1}}\{\mathbf{P}_{\hat{\mathbf{e}}\hat{\mathbf{e}}}\}$$

More on the notations of equations (13) and (14) as well as the use of MIMOST in Geodesy can be found in Andritsanos et al. (2000).



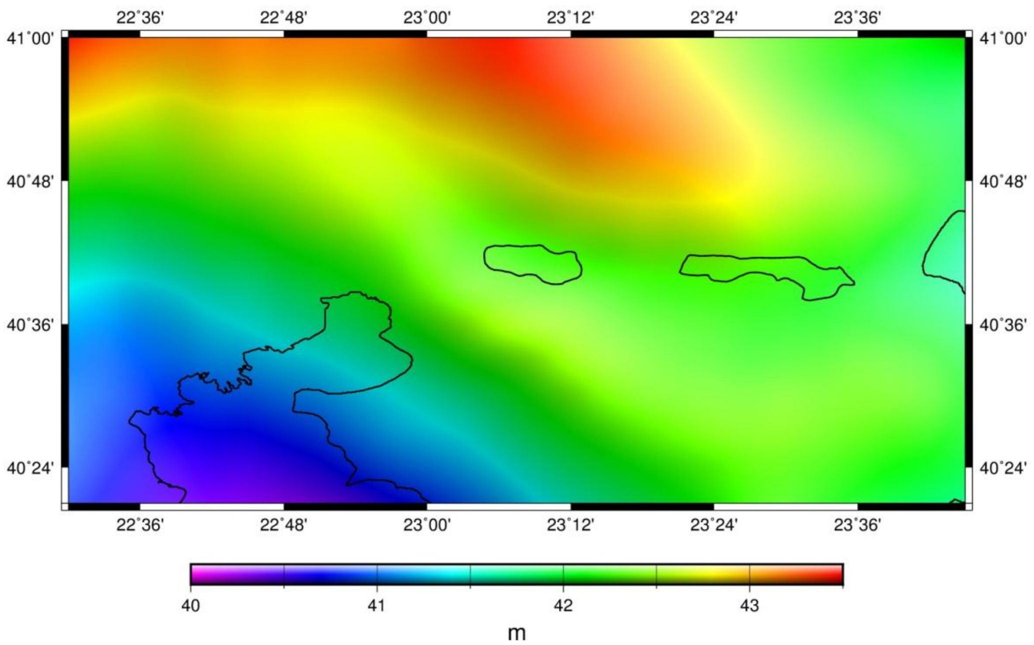


Figure 7. Combined MIMOST estimated geoid for Attica (top) and Thessaloniki (bottom).

The final combined MIMOST estimated geoid models for Attica and Thessaloniki are presented in Figure 7. The latter geoid solutions contain information from both the modified GOCE-DIR5 GGM as well as the GPS/levelling signal (see also Figure 6). The combination of both signals is based on the minimization criterion of the output error and due to this fact the sensitivity of MIMOST to the input noise is minimal (Andritsanos and Tziavos, 2012).

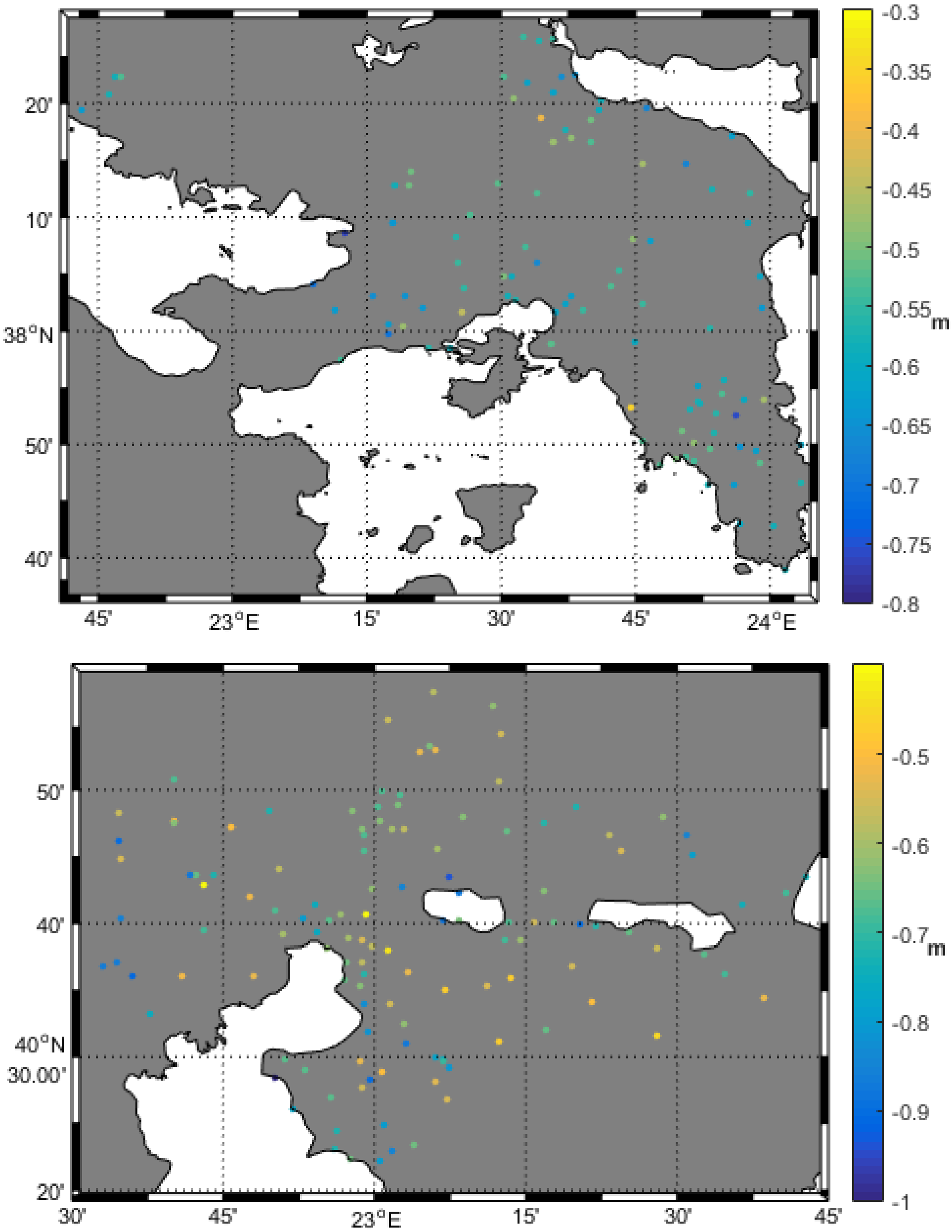


Figure 8. Distribution of the GPS/levelling data for Attica (top) and Thessaloniki (bottom). The colored values depict differences between geoid heights from combined MIMOST solution and those derived from GPS/levelling data.

The computed MIMOST combined geoid heights were then compared to the GPS/leveling derived geoid heights. In Table 3 are provided the statistical values of the differences between the geoid heights obtained from GPS/leveling and the corresponding geoid heights from a) the synthetic DIR-R5-EGM08 geoid model and b) the MIMOST solution. By examining the results of Table 3, the differences with the MIMOST solution present an improvement of 1.3 cm in Attica and 3.5 cm in Thessaloniki in terms of standard deviationwith respect to those with the DIR-R5-EGM08 model. Additionally, the range of the differences has also been reduced by 10 cm in Attica and 44 cm in the region of Thessaloniki.

Table 3. Statistics of geoid height differences for the areas of Attica and Thessaloniki between GPS/levelling geoid heights and a) GGMs geoid heights and b) combined MIMOST geoid heights.

	$N_{GPS/lev.} - N_{DIR-R5(140)+EGM08}$			$N_{GPS/lev.} - N_{comb}$		
Area	**mean[m]**	**std[m]**	**range[m]**	**mean[m]**	**std[m]**	**range [m]**
Attica	-0.406	0.080	0.514	-0.572	0.067	0.412
Thessaloniki	-0.488	0.160	1.014	-0.659	0.125	0.632

In Figure 8, the distribution of the GPS/levelling in Attica and Thessaloniki is depicted as well as the differences between GPS/levelling geoid and MIMOST geoid heights. Considering Figures 4 and 5 one can identify the previously described improvement in the geoid height differences using the MIMOST geoid solution for both study areas.

5. Conclusions

The extensive evaluation of the latest GOCE, GOCE/GRACE and combined GGMs have been carried out using GPS/levelling benchmarks at two regions, one in Central (Attica) and another in Northern (Thessaloniki) Greece. Local parametric models have been tested in order to remove the inherent datum inconsistencies, between the ellipsoidal, orthometric and geoid heights. Six parametric models have been selected and the GGMs signals has been used to its maximum power, as well as to lower degrees given the truncation of the spherical harmonic expansion. The GOCE/GRACE GGMs signal has been filled-in with EGM08 up to its maximum degree and order of expansion, representing the high frequency content of the gravity field spectrum. The 5th release of GOCE models estimated by the Direct as well as the Time-Wise approach and filled by EGM08 signal outperformed any other case, in terms of the standard deviation and the range of the differences at GPS benchmarks. A third order polynomial improved the results of the differences by 1 cm in the Attica and 2 cm in the Thessaloniki area, in terms of standard deviation. The truncation to degree 140 was selected based on various tests and previous studies. The 5th release of GOCE data showed an improvement at the low frequency band of the gravity spectrum when compared with GPS/levelling data at benchmarks. This improvement can be identified considering the statistics of the differences even before any parametric model application. The latter EIGEN-

6C4 GGM gave slightly worse statistics (3 mm in terms of std of the differences) than the synthetic GOCE(140)+EGM08 model.

The incorporation of GPS/levelling signal to the final combined geoid is feasible through MIMOST. A combined GPS/levelling/GGM geoid model using the geoid information from GOCE DIR-R5 to a degree 140 and EGM08 residual signal has been estimated. The combined geoid contains a minimal effect of datum inconsistencies, since the coupling of GGM and GPS/levelling geoid heights is based on a specific minimization criterion. The comparisons showed an improvement of 1.3 cm in Attica and 3.5 cm in Thessaloniki in terms of the standard deviation of the differences before any fit. An even greater improvement was observed in the range of the differences: 10 cm in Attica and 44 cm in Thessaloniki region.

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