Comparison of various GPS processing solutions towards an efficient validation of the Hellenic vertical network: The E.LE.V.A.T.I.ON project

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Abstract

A research project for the validation of the Hellenic vertical network is currently in progress. Two investigation areas in Central and Northern Greece have been chosen. The areas include several benchmarks of the national trigonometric and leveling networks. Static Global Positioning System (GPS) observations as well as classical terrestrial leveling are performed to assess the internal accuracy of the two networks. Some numerical tests based on GPS and leveling measurements are presented and the goals of the project are outlined. The strategies followed in the processing of GPS data are presented with emphasis on their future use to the project evolution. GPS observations have been processed using various commercial as well as scientific software packages in order to examine the influence of the processing algorithms to the final results. Significant differences between the results of the various software

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packages have been revealed, particularly in the case of challenging observation conditions. Finally, comparisons of the estimated geoid heights at GPS benchmarks (BMs) to EGM2008 geoid information are performed as a first step towards the evaluation of the Hellenic vertical network. These comparisons indicate that the two investigations areas are of different internal accuracy namely 8.3 cm and 15.8 cm in terms of sd of the differences at Attica and Thessaloniki test areas.

1. Introduction

The determination of the 3-D positions is feasible nowadays with particularly high accuracy using modern Global Navigation Satellite System (GNSS) positioning techniques. In contrast, the determination of vertical positions is much more demanding, mainly due to the inherent connection between the vertical reference systems and the earth's gravity field. Height information, reckoned from an equipotential surface, is of particular importance for a variety of applications from coastal management to construction and monitoring of technical works like highways, railways, metros and bridges.

A project for the validation and quality control of the Hellenic vertical network named **E.LE.V.A.T.I.ON** (**E**valuation of the He**LE**nic **V**ertical network in the fr**A**me of the European sys**T**ems and control networks **I**nterconnecti**ON** – Application in the areas of Attica and Thessaloniki) is currently in progress. Two investigation areas, one in Attica (Central Greece) and another in Thessaloniki (Northern Greece) have been chosen. The areas include several height benchmarks (BMs) of the national trigonometric and leveling networks. Static GPS observations as well as classical spirit leveling in combination with trigonometric leveling are performed to assess the internal accuracy of the two networks. Numerical tests based on GPS and leveling measurements are presented and the goals of the project are discussed. The control and re-evaluation of the Hellenic vertical network is the main objective of the proposed project. Height information of high accuracy and reliability in a common reference system is essential. Especially today, with the pan-European effort for the establishment of a common European Vertical Network (Sacher et al., 2007), the validation of the Hellenic network seems a prudent decision. In order to underline the importance of reference system unification, it should be mentioned that the International Association of Geodesy (IAG) established a Special Study Group (SSG) for the connection of various reference systems in Europe. This SSG (EUREF – http://www.euref.eu) since 1989 has introduced the European Terrestrial Reference System of 1989 (ETRS89). The connection of the Hellenic 3-D network with ETRS89 has been established through the Hellenic Positioning System (HEPOS). HEPOS is a nation-wide Real Time Kinematic (RTK) network based on 98 reference stations established for the modernization of the geodetic infrastructure of Greece (Gianniou, 2008). During the next years the connection of the vertical datum with Europe has to be done; this is also a European Community directive under the name "INSPIRE". Before the connection, the validation of the vertical network has to be carried out.

The first-order vertical control network of Greece was established and measured by the Hellenic Army Geographic Service from 1963 to 1986 (Milona – Kotroyanni, 1989). Approximately 11000 km of traverses and 11000 vertical control benchmarks are the characteristics of Greek vertical network. The tide gauge in Piraeus harbor is the fundamental point of the network. On the other hand, the first order Hellenic trigonometric network has some height information, due to some trigonometric leveling lines. This vertical information has not been validated since its creation. The validation of the vertical reference network before the establishment of the European interconnection is thus essential.

The European committee for the continental control networks works under the auspices of the European Council on the measurement and establishment of both a horizontal as well as a vertical European reference system. A Vertical System is characterized by its Datum (point of reference) and the type of height used. The Datum point is estimated by the Mean Sea Level (MSL) in the area, as determined by tide gauge measurements. In Europe tide gauges exist in various regions: in the Baltic, in the North Sea, in the Mediterranean, in the Black Sea and in the Atlantic Ocean. Level differences between various tide gauges can reach several centimeters. In addition, national vertical datum points are based on historical facts and not always referenced to the MSL, e.g., the zero-point of the Amsterdam tide gauge is defined as the mean high tide in the year 1684.

Another issue is the use of various types of heights around Europe. Thus, orthometric heights are used in Belgium, Denmark, Italy, Greece, etc. and normal heights are used in France, Germany, Sweden and the Eastern European countries. In 1945 the integration of national systems started, while the establishment of a common system around Europe was divided to various solutions in Western and Eastern Europe due to political reasons.

Greece, in particular, has not been connected yet with any of the unified vertical reference systems. As a consequence, difficulties arise in planning and executing cross-border works like roads, railways and pipeline constructions. A prerequisite for the Hellenic vertical datum connection is its evaluation. The validation of the height data must be based on the interpretation of the inner accuracy of the solution and the external control using independent data.

The first stage of the ELEVATION project is dedicated to the compilation/validation of existing data and the collection of new observations. These observations were collected during the first stage of the project (August-October, 2012) and referred to the update and enrichment of the existing GPS and leveling database. GPS observations near leveling benchmarks (reperes) of the Greek vertical network as well as on trigonometric pillars were collected. The connection between various benchmarks using classical spirit or trigonometric leveling with simultaneous reciprocal-observations is also part of the first stage of the project. The second stage of the project is based on the data processing. GPS observations have been processed using various commercial as well as scientific software packages in order to examine the influence of the processing algorithms to the final result.

2. Theoretical background on heights

The need to separate horizontal and vertical positions stems from the different accuracy provided by terrestrial observations. Horizontal directions are measured with increased accuracy compared to the vertical ones. This is due to the atmospheric refraction effect. The abovementioned fact introduces greater uncertainty to vertical positioning. This is why classical geodetic observations are divided into horizontal directions and distances for horizontal positioning and spirit leveling measurements for vertical positioning (Torge, 2001).

Height data are referenced to suitable level surfaces, which represent characteristic elements of the observation environment. Heights are connected with human activity and thus their link with physical characteristics is necessary. A characteristic surface is the Mean Sea Level (MSL). This surface represents the traditional connection of all human activities with the natural environment. Practically, it is common knowledge that the MSL is a zero-height surface. Theoretically speaking, MSL in a global scale constitutes a balance surface of waters and, excluding the presence of the quasi-stationary Dynamic Ocean Topography, represents an equipotential surface of Earth's gravity field. In this manner, the concept of geoid as a height reference surface is introduced. The geoid is an equipotential surface of the Earth's gravity field that to a first approximation coincides with the MSL in global scale, provided that the effects of tides and ocean currents are removed. In a well-defined national vertical control network, heights are referenced in a datum point of zero altitude. Usually, the zero-height point is defined by local MSL observations from tide gauge records. In reality, the sea-level change is measured from a conventionally selected level, which is considered constant: the tide gauge zero.

Another reference surface used is the ellipsoid of revolution. The ellipsoid is not a physical surface and is used only as a model of the Earth’s surface for horizontal positioning, due to the simplicity of its mathematical description. Data from geodetic satellite missions can be referenced to an ellipsoid of revolution. The data of such missions will be used for the validation of the current vertical network. The main height reference surfaces used in this work are depicted in Figure 1 and analytically described in the methodology section.

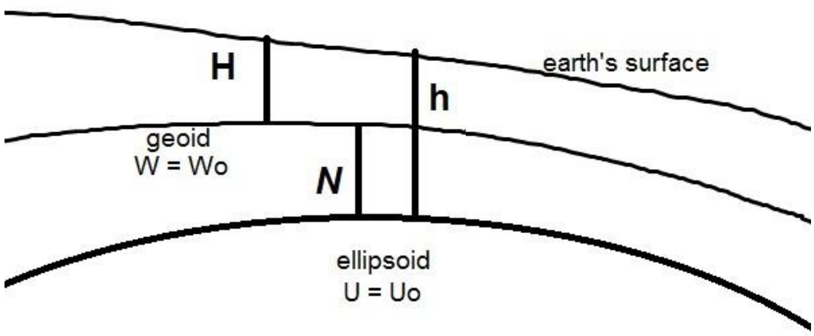

Figure 1: Height reference surfaces

A point in space can be identified using three coordinates: the latitude, the longitude and the height. The horizontal coordinates are referenced to the surface of a model ellipsoid of revolution which is a geometrical-mathematical surface related to the MSL in local or global scale. The height of a point P can be referenced along the vertical on the ellipsoid and is called ellipsoidal height h_P.

However, in geosciences the altitude of a point must be referenced to the MSL, or more precisely to the vertical reference system. As it is known, the ellipsoidal model does not coincide with the MSL but has a deviation from -100 to 100 m, globally. The dependence of the vertical reference system from the gravity field seems obvious, since an equipotential surface of this field is the first approximation of the MSL at a global scale.

The earth gravitational potential is the potential of the attracting masses including the atmosphere and can be expressed in spherical harmonic expansion (Hofmann-Wellenhof and Moritz, 2005):

$$V_e(r,\theta,\lambda) = \frac{GM}{R}\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left(\frac{R}{r}\right)^{n+1} C_{nm}\bar{Y}_{nm}(\theta,\lambda), \tag{1}$$

where (r,θ,λ) are the spherical coordinates of the computation point, GM is the product of Newton's gravitation constant and Earth's mass (including the atmosphere), R is the mean radius of the Earth, C_{nm} a constant coefficient of degree n and order m, $\bar{Y}_{nm}$ are the fully normalized spherical harmonic functions:

$$\bar{Y}_{nm}(\theta,\lambda) = \begin{cases} \bar{P}_{nm}(\cos\theta)\cos|m|\lambda, & m \geq 0 \\ \bar{P}_{n|m|}(\cos\theta)\sin|m|\lambda, & m < 0 \end{cases}, \tag{2}$$

and $\bar{P}_{nm}$ are the fully normalized Legendre functions of the first kind. The above expression is the solution of the boundary value problem for the potential and it is valid for any point outside the Earth masses, in which space Laplace's equation is applied (Martinec, 1998).

Due to Earth's rotation, the gravitymeters measure additionally a centrifugal acceleration which leads to the centrifugal potential:

$$\Phi(r,\theta) = \frac{1}{2}\omega_e^2 r^2 \sin^2\theta, \tag{3}$$

where ω_e is the angular velocity of Earth's rotation. Therefore the gravity potential can be expressed as:

$$W(r,\theta,\lambda) = V(r,\theta,\lambda) + \Phi(r,\theta). \tag{4}$$

The connection between geometrical and physical characteristics is established following the equation:

$$\mathbf{g} = \nabla W \tag{5}$$

and the magnitude of the gravity vector:

$$|\mathbf{g}| = g = -\frac{dW}{dn}, \tag{6}$$

where dn is the differential length along the plumb-line.

As it is already mentioned, a height reference surface must be related with the physical environment through an equipotential surface of Earth's gravity field, a surface of constant value of W. Especially, the surface $W=W_0$, which is approximated by the MSL is known as the geoid. Therefore, the height "above MSL" is defined precisely as the height "above the geoid". Let $P_0^{(j)}$ a point near a tide gauge, with a gravity potential $W_0^{(j)}$. There exist three different kinds of heights depending on the potential definition at the point of interest. This potential difference is known as geopotential number:

$$C_P^{(j)} = W_0^{(j)} - W_P, \tag{7}$$

where W_P is the gravity potential of point P on Earth's surface. The geopotential number is a unique characteristic of the space domain and using a scale factor of normal gravity γ_0 can be expressed as height coordinate:

$$H_P^{dyn(j)} = \frac{C_P^{(j)}}{\gamma_0}, \tag{8}$$

which represents the dynamic height of point P (related with the local vertical datum j). The normal gravity scale factor is usually taken equal with the magnitude of normal gravity computed at a mean latitude ($\gamma_0 = 9.806199203$ m/s^2). It is noted that the dynamic height is expressed in length units and can be used as a height. Nevertheless, it does not provide any geometrical information: it is just a physical quantity – the potential related to the geoid surface.

Seeking for a geometrical definition the integration of equation (8) is performed:

$$W_P = W_0^{(j)} - \int_{P_0^{(j)}}^{P} g dn . \qquad (9)$$

Using the geopotential number at point P it becomes:

$$C_P^{(j)} = \int_{P_0^{(j)}}^{P} g dn . \qquad (10)$$

Equation (10) shows the relation between the geopotential numbers, gravity and measurements of vertical difference between equipotential surfaces along the plumbline. The vertical differences (in length units) are observed using classical spirit leveling. Taking the measured track always perpendicular with the equipotential surfaces (plumb-line), the geopotential number can be computed as:

$$C_P^{(j)} = \int_{\overline{P}^{(j)}}^{P} g dH , \qquad (11)$$

where dH is the unit-length along plumb-line and $\overline{P}^{(j)}$ is located at the intersection of the plumb-line with the geoid surface. Solving the above equation along the vertical length, $H_P^{(j)}$, called orthometric height, one can write:

$$H_P^{(j)} = \frac{C_P^{(j)}}{\overline{g}_P^{(j)}} , \qquad (12)$$

where

$$\overline{g}_P^{(j)} = \frac{1}{H_P^{(j)}} \int_{\overline{P}^{(j)}}^{P} g dH \qquad (13)$$

is the average gravity along the plumb-line. In this specific case, a density model for the masses inside the Earth is needed. This fact dictates the direct dependence of orthometric height accuracy with the accuracy of the density model used.

The relation between ellipsoidal heights, measured from GNSS and orthometric heights is:

$$h = H + N \, , \tag{14}$$

where h is the ellipsoid height along the vertical on the model surface, H is the orthometric height from the geoid surface, measured along the plumb-line and N is the geoid undulation (distance from the geoid to the ellipsoid) along the vertical on the ellipsoid. According to the definition, the orthometric height is independent of the ellipsoid model used. However, the geoid undulation is based on the ellipsoid choice because it is expressed as the difference from a specific model. Geoid heights can be derived using local gravity information in combination with global features provided by a geopotential model. The most recent global geopotential model calculated from a special spectral combination of terrestrial and satellite data is EGM2008 (Pavlis et al., 2012).

3. Data collection and analysis

3.1. GPS measurements

The main purpose of the GPS measurements was the determination of the ellipsoidal heights of the trigonometric and leveling BMs. As known, leveling BMs are often established on vertical elements like walls or columns and, thus, they are not adequate for GPS measurements. In such cases, we established new points offering good satellite visibility on sites as close as possible to the original BMs (distances up to 200 m). These newly established points were connected to the original BMs by means of double-run spirit leveling. Figures 2 and 3 depict the location of the trigonometric and leveling benchmarks in Attica and Thessaloniki, respectively.

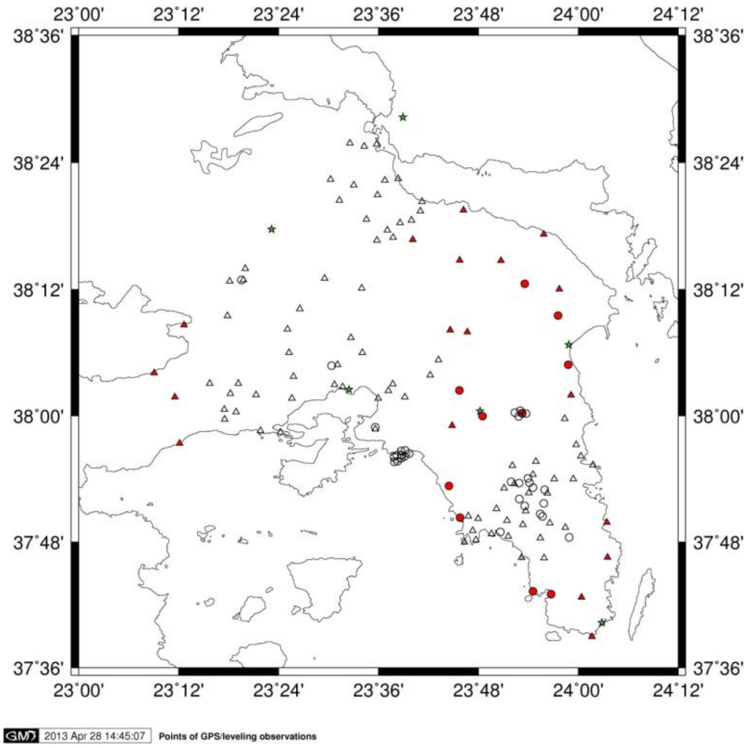

Figure 2: Points in Attica region. Star: HEPOS stations, Circle: previously measured height BMs, Solid Circle: newly measured BMs, Triangle: previously measured trigonometric BMs, Solid Triangle: newly measured trigonometric BMs.

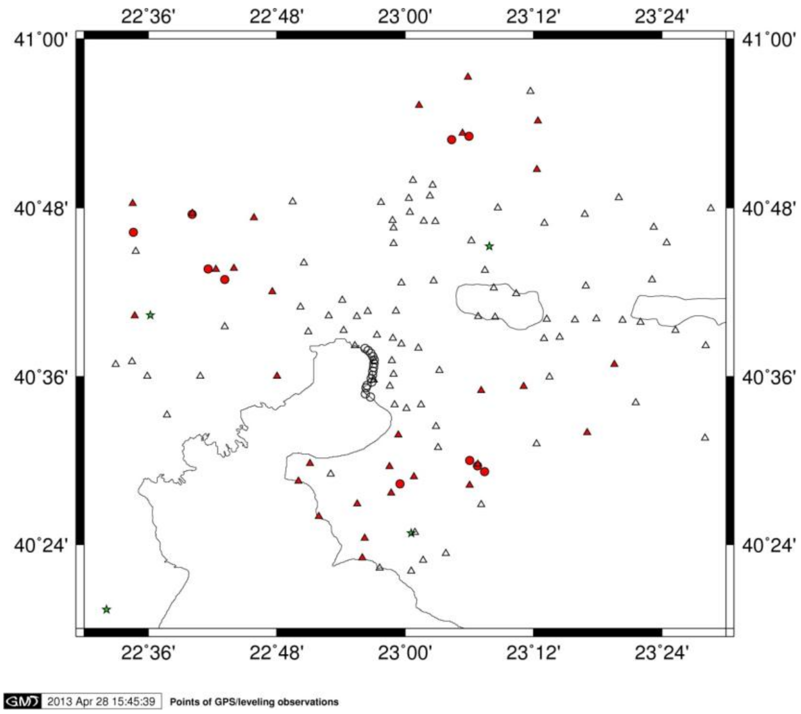

Figure 3: Points in Thessaloniki region. Star: HEPOS stations, Circle: previously measured BMs, Solid Circle: newly measured BMs, Triangle: previously measured trigonometric BMs, Solid Triangle: newly measured trigonometric BMs.

In order to ensure high accuracy in the determination of the ellipsoidal heights of the BMs, the GPS measurements have been designed carefully. A key parameter for this work was the selection of an adequate geodetic reference frame. The latest International Terrestrial Reference Frame (ITRF2008) would be the best choice, as it ensures the highest possible accuracy. However, this solution would require the connection of the BMs to permanent reference stations with well known ITRF coordinates, i.e. IGS and/or EUREF/EPN stations. As there are no IGS stations in Greece (except two proposed stations in Athens and Chania), the length of the baselines to the closest IGS stations would be of the order of hundreds of kilometers imposing observation times of at least 24-48 hours, which was improper for our project. Regarding the EPN stations, NOA1 in Athens and AUT1 in Thessaloniki are situated within the two project areas at distances up to 60 km away from the BMs. However, we wanted to have every point connected to at least two stations, which would lead to baseline lengths of the order of hundreds of kilometers. So, instead of using EPN stations, we preferred to use stations of the Hellenic Positioning System HEPOS (Gianniou, 2008). The system consists of a dense network of stations, offering the possibility to connect each BM to two stations, while keeping short baseline lengths. The baselines measured in the area of Attica and Thessaloniki are shown in Figures 4 and 5, respectively. The corresponding mean baseline length for each area was 20.9 km and 23.4 km, respectively. The maximum baseline length was 44 km and only four vectors among a total number of 134 baselines exceed 40 km. Given the aforementioned baseline length the rapid-static method could have been used. However, in order to increase the accuracy of the results, the static method has been used adopting a minimum occupation time of 1 hour at each point. The measurements have been conducted using dual frequency receivers, i.e. Topcon HiperPro in Attica and Leica SR520 in Thessaloniki. The logging interval was 15 s and the elevation mask 10^{o}. The antenna heights were measured with an accuracy of ±1 mm. More details about this campaign can be found in Anastasiou et al., 2012. Table 1 summarizes the number of trigonometric and leveling BMs in the two areas of the ELEVATION project. Figures 4 and 5 depict the location of the benchmarks and the HEPOS stations used for the processing of the baselines in Attica and Thessaloniki respectively.

Table 1: Trigonometric and leveling benchmarks used in the project.

	Trigonometric BMs		**Leveling BMs**	
	Old measurements	New measurements	Old measurements	New measurements
Attica	80	20	7	8
Thessaloniki	92	29	-	10

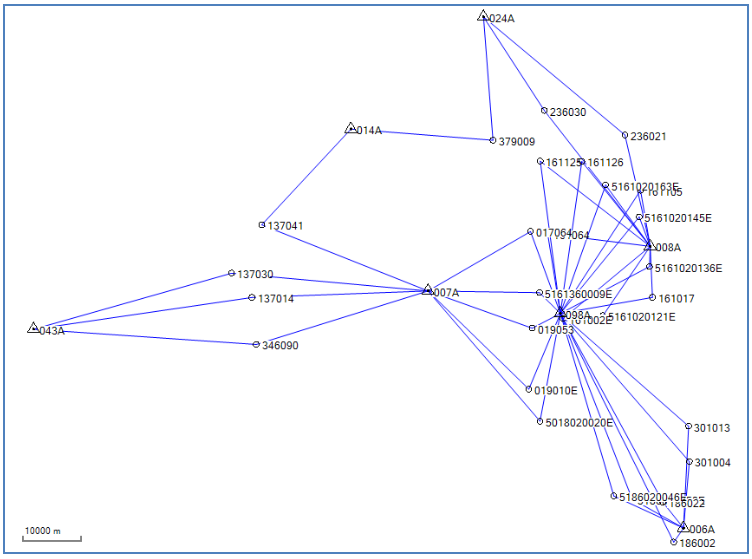

Figure 4: GPS Baselines measured in the area of Attica. Triangle: HEPOS station, circle: newly measured BMs

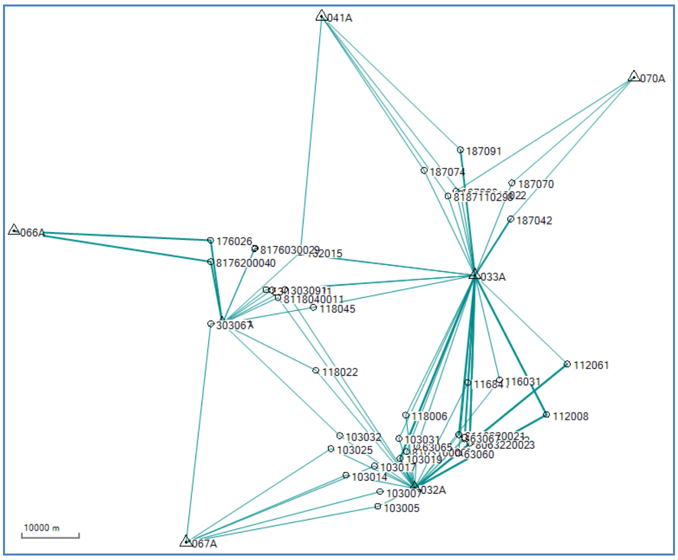

Figure 5: GPS Baselines measured in the area of Thessaloniki. Triangle: HEPOS station, circle: newly measured BMs

3.2. Leveling observations

Given the availability of a number of GPS/Leveling benchmarks with collocated GPS and leveling observations, the first step was the selection of new benchmarks (BMs) to be measured. The new BMs were selected from the National Trigonometric and Leveling Network, established by the Hellenic Military Geographic Service (HMGS) in order to guarantee the connection to the national horizontal and vertical networks. Of the total number of height benchmarks that were found after the research in the two investigation areas only a part of them were chosen for conducting the leveling measurements. In order to reassess the leveling network in the investigation areas of Attica and Thessaloniki a combination of ground based techniques were used for the determination of orthometric height differences. The two types of techniques that were applied are the classical spirit leveling and the special trigonometric leveling. More on the techniques used for the leveling observations as well as results and comparisons from the evaluation procedure can be found in Anastasiou et al. (2012).

4. Data processing and results

4.1. GPS data processing schema

Due to the challenging for GPS measurement environment at some BMs (foliages, obstacles, electromagnetic interferences), difficulties in the data processing had been expected. In order to have a better control on the quality of the results, it was decided to perform independent computations using five different software packages available at the three Institutions participating in the research project. In that manner it would be possible to make an extended comparison of the used software packages. Table 2 summarizes the programs used and their characteristics.

Table 2: Software packages used for the GPS data processing.

Software	Version	Release year	Manufacturer	Abbreviation in paper
Bernese	5.0	2007	University of Bern	BERN
Geomax Geo Office	2.0	2009	Geomax	GGO
GrafNet	8.40	2012	Novatel (XYZ solutions)	GN
Trimble Business Center	1.12	2007	Trimble	TBC
Topcon Tools	7.5.1	2010	Topcon	TT

For the processing with Bernese, IGS precise orbits have been used. For the processing with the commercial software packages we used broadcast orbits. The error in the baseline length introduced by the orbital error can be approximated by the formula (Teunissen and Kleusberg, 1998):

$$\frac{db}{b} = \frac{dr}{r} \tag{15}$$

where *db/b* is the relative baseline error and *dr/r* the relative orbital error. Given that the maximum baseline length was 44 km and assuming an orbital error of 2 m, it comes out that the maximum error in the baseline length due to the orbital error did not exceed 4 mm, which is fully sufficient for our purposes.

With Bernese the processing parameters described in the CODE Analysis strategy (ftp://ftp.unibe.ch/aiub/CODE/0000_CODE.ACN) were used. For fixing the ambiguities, the SIGMA algorithm (Dach et al., 2007) has been used together with the L1/L2 method (for baselines up to 20 km) and the widelane/narrowlane method (for longer baselines). With the commercial software packages the default processing parameters of each software have been used.

4.2. GPS data processing results

During the baseline processing we encountered certain difficulties due to the aforementioned unfavorable satellite signal reception at some BMs. For several baselines the initial processing (i.e. using all observed satellites) yielded fixed solutions but with poor statistics (flagged fixed), whereas for a limited number of baselines the initial result was a float solution. In order to improve the results we reprocessed these baselines after rejecting observations with large residuals. In this way, most of the flags were removed and most of the float solutions became fixed. This procedure was followed with the commercial programs, which are suitable for such kind of interventions. In contrast, such intrusions are quite complex in Bernese, so no similar attempts were made with this software. Table 3 summarizes the initial and final results obtained from each processing software.

Table 3: Baseline solution results.

Initial Solution	Final Solution	Comments	# of baselines per software				
			BERN	**GGO**	**GN**	**TBC**	**TT**
Float	Float	Solution not used	6 *	1	-	-	-
Float	Fixed	Fixed obtained after deactivating satellites	- *	3	-	6	7
Fixed Flagged	Fixed	Solution improved by deactivating satellites	- *	10**	3***	19	19
Fixed	Fixed	Minor or no-interventions	128	120	131	109	108

(*) *Using Bernese no attempts were made to improve the initial solutions.* *(**) GGO does not flag weak baselines; the averaging limit was used instead.* *(***) For six flagged baselines it was not possible to obtain non-flagged solution.*

The results of the different software programs agreed quite well in the case of BMs that offer good observation conditions. On the contrary, for baselines involving BMs with unfavorable signal reception, significant differences resulted between the solutions of the different programs. For this reason, the comparison of the different software packages was done distinguishing between two classes of baselines: typical and problematic baselines. Two criteria were used for the classification of the baselines. The first criterion was the statistics of the solutions, i.e. the a-posteriori reference variance, the RMS and the standard error of the baseline components. Additionally, for the Bernese solutions we used as an additional criterion the percentage of resolved ambiguities, which is being reported. The second criterion for the classification of the baselines was the closure error. Instead of using loop closures we computed the closure error based on the difference between the coordinates resulted for each BM from each one of the two baselines available for that BM (from the two nearest HEPOS stations). For our dataset this approach of computing the closure errors is considered to yield more realistic results compared to loop closures, for two main reasons. First, the two baselines used for each closure check are uncorrelated. If we had solved also the baseline between the two HEPOS stations, each triangle would consist of three correlated vectors. As known, three receivers measuring in parallel produce only two stochastically uncorrelated baselines (Hofmann-Wellenhof et. al, 2008). Secondly, the baseline between the HEPOS stations was in some cases twice as long as the baselines to the BMs, e.g. 69 km between stations 043A and 007A (Figure 6). Baselines of such length cannot be precisely estimated from occupations of one-hour duration. This would lead to increased loop closure errors. In our study the horizontal closure error (dS) is:

$$dS = \sqrt{(E_{RS\_1} - E_{RS\_2})^2 + (N_{RS\_1} - N_{RS\_2})^2} \qquad (16)$$

where the indexes RS_1 and RS_2 denote the coordinates obtained from the baselines from the nearest and the next nearest HEPOS reference station (RS), respectively. For the vertical closure we used the absolute value of the difference between the ellipsoidal heights obtained from each pair of baselines, i.e.:

$$|\boldsymbol{dh}| = |\boldsymbol{h}_{RS\_1} - \boldsymbol{h}_{RS\_2}| \qquad (17)$$

Using the aforementioned criteria 39% of the baselines in Attica (11 among 28 baselines) and 23% of the baselines in Thessaloniki (9 among 39 baselines) have been designated as problematic. Tables 4 and 5 give the mean and maximum values of the horizontal and vertical closure errors for the typical and problematic baselines in Attica and Thessaloniki, respectively. Float solutions have been excluded from the computation of the results. The mean values are depicted graphically in Figure 6 (Attica) and Figure 7 (Thessaloniki). In Addition, these figures include statistics computed over the entire sample of baselines for each area. The reason for this is that the problematic baselines were not common among the different software packages. Thus, the only way to directly compare the results is to examine the statistics over the same sample (all baselines). Comparing the results for the two areas it becomes obvious that the baselines in Thessaloniki offer slightly lower accuracy, on the order of few mm to 1 cm, compared to that of Attica. This is why we present our results distinguishing between the two areas. The lower performance in Thessaloniki can be mainly attributed to the fact that the measurements have been conducted with receivers of older technology (Leica SR520) compared to the receivers used in Attica (Topcon Hiper Pro).

Table 4: Statistics of the horizontal closure error (values in m).

	Attica				**Thessaloniki**			
	Typical baselines		**Problematic baselines**		**Typical baselines**		**Problematic baselines**	
	Mean	Max	Mean	Max	Mean	Max	Mean	Max
BERN	0.028	0.060	0.075	0.120	0.054	0.152	0.108	0.205
GGO	0.012	0.031	0.015	0.045	0.037	0.117	0.045	0.125
GN	0.024	0.072	0.059	0.132	0.038	0.200	0.118	0.257
TBC	0.010	0.019	0.021	0.043	0.020	0.056	0.029	0.070
TT	0.012	0.027	0.034	0.104	0.020	0.050	0.062	0.140

Table 5: Statistics of the vertical closure error (values in m).

	Attica		**Thessaloniki**	
	Typical	**Problematic**	**Typical**	**Problematic**

	baselines		baselines		baselines		baselines	
	Mean	Max	Mean	Max	Mean	Max	Mean	Max
BERN	0.057	0.184	0.101	0.204	0.062	0.169	0.149	0.385
GGO	0.040	0.074	0.034	0.074	0.033	0.115	0.039	0.112
GN	0.048	0.202	0.072	0.185	0.052	0.162	0.101	0.239
TBC	0.038	0.085	0.040	0.075	0.040	0.098	0.049	0.155
TT	0.041	0.094	0.060	0.168	0.041	0.122	0.066	0.146

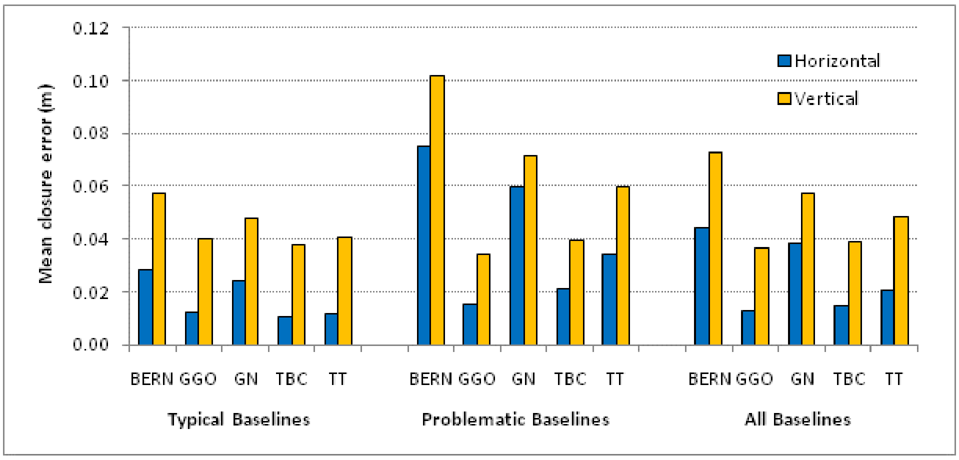

Figure 6: Mean horizontal and vertical closure errors for the baselines in the area of Attica.

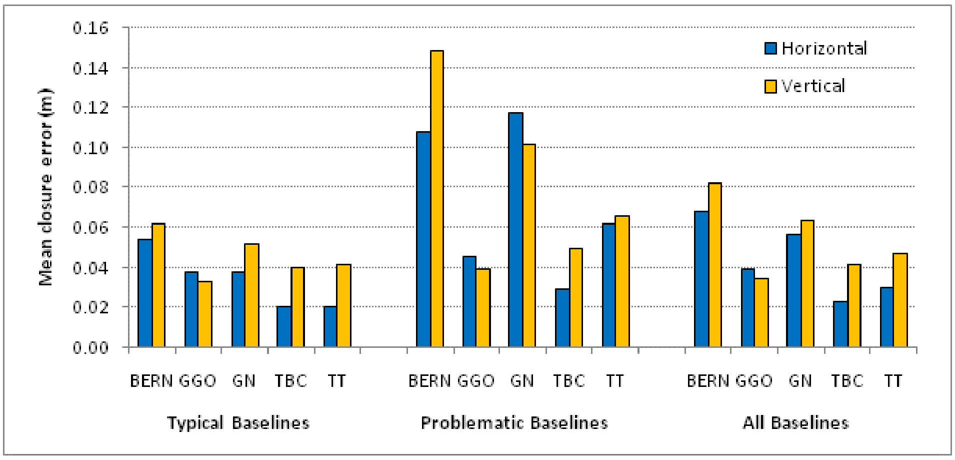

Figure 7: Mean horizontal and vertical closure errors for the baselines in the area of Thessaloniki.

In order to allow for a comparison of the closure errors of the different software we computed the ratio of the mean closure error of each software to the respective error of the best performing software. This comparison has been done separately for each group of baselines (*typical*, *problematic* and *all*) as well as for the horizontal and the vertical error. Figures 8 and 9 give the computed ratios for the baselines in Attica and Thessaloniki, respectively. The best performing software can be easily recognized as its ratio equals 1. In each figure six bars are pointing at 1: for each group of baselines, one bar for the horizontal and one for the vertical error. In the case of Figure 9 within each group of baselines the lowest horizontal and vertical errors were obtained from the same software (TBC for the typical, GGO for the problematic and GGO for all baselines). On the contrary, in the case of more noisy observations in Thessaloniki (Figure 9) the best performance in horizontal and vertical closures within each group of baselines was achieved by different software.

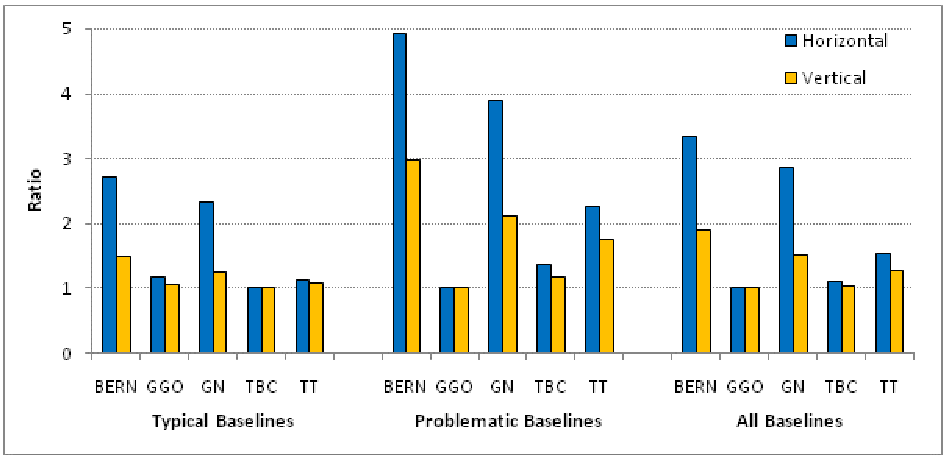

Figure 8: Ratio of mean closure error of each software w.r.t. the best performing software, computed separately for dS, |dh|, typical, problematic and all baselines in Attica.

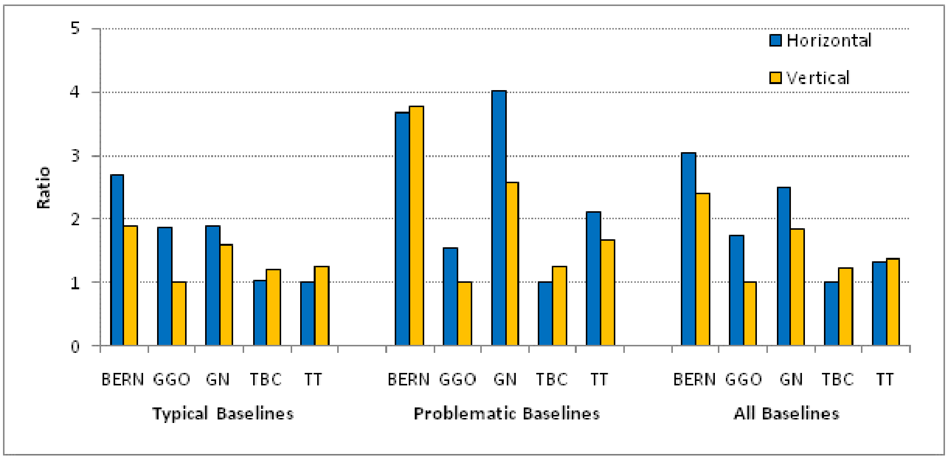

Figure 9: Ratio of mean closure error of each software w.r.t. the best performing software, computed separately for dS, |dh|, typical, problematic and all baselines in Thessaloniki.

4.3. Discussion of GPS results

Before discussing the results of the used software packages, we would like to stress that our purpose was not the assessment of the relative performance of the various software programs. Such comparisons require a much larger data set of baselines and the use of the latest versions of all programs, which was not the case in our study (see also Table 2 for the release year of each software). Actually, our goal was to demonstrate the importance of the processing software in the accuracy of the results, especially in the case of problematic baselines.

Examining the overall relative performance of the five software (Figures 6-7: columns *all baselines*) it becomes obvious that the four commercial software packages yield better results compared to Bernese. Of course, this conclusion does not reduce the worth of this well-acknowledged software, which undoubtedly belongs to the best scientific GNSS processing software worldwide. One should keep in mind that Bernese mainly focuses on the processing of measurements of long duration (e.g. daily occupations) collected at sites offering good observation conditions (e.g. reference stations) over long distances (baseline length of the order of several hundreds or thousands of kilometers). The detailed modeling of many errors sources (ocean, atmospheric and solid earth tidal displacements, earth orientation variations, satellite phase center offsets and patterns etc.) (Dach et al., 2007) is necessary for long baselines**,** but does not actually improve the solution of short baselines, as these errors cancel-out when forming double-differences. In addition, the long duration of the observations is important for Bernese in order to perform realistic estimations, e.g. for the tropospheric delay. On the other hand, commercial software packages are designed to process not only data of good quality, but also problematic measurements collected under unfavorable field conditions.

Among the four commercial programs GrafNet yielded more noisy results. GrafNet is part of NovAtel's GNSS post-processing software package, which is well-acknowledged for GrafNav, a kinematic baseline and Precise Point Positioning (PPP) processor based on a Kalman filter. GrafNav and GrafNet use the same GNSS processing engine. This processing engine is proven to provide great results for kinematic measurements (Dao et al., 2004; Bláha et al., 2011). Examining Figures 6-7 (columns *typical* and *problematic baselines*) one may conclude that for static observations of good quality, GN yields somewhat worse results compared to the other commercial software packages, but in the case of problematic baselines the results were up to 4 times worse. This could be attributed to the processing engine, which is by design more suitable for kinematic measurements. Examining Figures 8-9 one may conclude that the performance of the three other programs is roughly on the same level. For example, GGO shows slightly better performance in the case of problematic baselines in Attica. On the other hand, one baseline in Attica could not be solved by GGO, a fact that is not reflected in the figure. In the case of problematic baselines in Thessaloniki TBC performs significantly better than GGO. Observing the columns *all baselines* in Figures 8-9, we can conclude that GGO, TBC and TT provide more-or less comparable results. If we consider jointly the results in both areas, TBC shows the best performance. We attribute this superiority mainly to the fact that TBC is the only software among the three programs that gives detailed baseline processing report that contains a graphical representation of the observation residuals. This functionality allows the detection and exclusion of noisy observations, which considerably improves the solution. According to its manual TT has the same capability, but it is available only if the "Advanced Module" for processing has been licensed (Topcon, 2009). Regarding GGO, one could expect that the graphical representation of residuals would be supported as this program is practically the same as LGO (Leica Geo Office). However, comparing the two software packages one can see that certain functionalities of LGO are not available in GGO (Leica, 2010).

Examining the closure errors of typical and problematic baselines (Figure 6) we can see that - although the horizontal errors are lower than the vertical- they increase up to 3 times in the case of problematic baselines (TT). On the contrary, the increase of the vertical errors is limited to a factor of 1.7 (BERN). For the sake of clarity, we would like to stress that in the case of problematic baselines the vertical errors are still larger. However, the accuracy degradation caused by the problematic observations is higher for the horizontal component. This is a result of practical importance for the professional surveyors, who often measure in difficult environments and they are mainly interested in the horizontal accuracy.

Figure 6 verifies the general rule, which states that the vertical accuracy of GPS baselines is considerably lower than the horizontal accuracy. Looking at Figure 7 we can find some exceptions to this rule. More specifically, GGO provided smaller vertical errors for all group of baselines in Thessaloniki and GN showed similar behavior in the case of problematic baselines. To some extend these results could be explained by the fact that the observations in Thessaloniki are characterized by increased noise, as discussed above. As explained in the previous paragraph, the relationship between horizontal and vertical precision alters in the case of problematic observations. However, even in the case of problematic observations the vertical errors still remain generally higher. Thus, the different behavior of GGO and GN is believed to originate from the particular processing algorithms implemented in each software package. This investigation requires detailed comparison of the different GNSS processing engines, a task that is beyond the scopes of this paper.

4.4. Comparisons with global geopotential model EGM2008

The initial stage of the validation at GPS/leveling benchmarks is based on comparisons with external information. GPS/leveling provides the geometric connection between different height systems (geometric/ellipsoidal and orthometric height). According to eq. (14) a geometric estimation of the geoid can be derived using ellipsoidal and orthometric height information. The determination of this "geometric" geoid is directly comparable to the "physical" one derived from a geopotential model of high accuracy and resolution. As presented in the theoretical part, the resolution of the geopotential model is based on the degree and order of its coefficients expansion and its accuracy on the commission and omission errors estimated during the adjustment process (Hofmann-Wellenhof and Moritz, 2005). It should be kept in mind though that such a "geometric" geoid model is of limited, if any, theoretical rigorousness. This is due to the fact that the formed $h - H$ differences do not realize the geoid, i.e., a physical surface of constant gravity potential (W_0). They rather realize the difference between the two heights along the vertical lines with any systematic distortions due to the different datum of h and H. The major problem of the established Greek vertical datum is its systematic distortion due to the largely unknown accuracy of the BM orthometric heights. The non sufficient documentation on the adjustment procedure (constraints type and number) and the lack of the covariance matrix estimation …………

The global geopotential model used in the comparisons is the state-of-the-art spherical harmonics expansion geoid model based on various data sources combined, Earth

Gravitational Model 2008 - EGM2008 (Pavlis et al., 2012). This model incorporates optimally surface gravity observations, satellite altimetry data and newly available products from gravity dedicated satellite missions (GRACE). The spherical harmonic expansion of EGM2008 reaches degree 2190 and order 2159, resulting in a spatial resolution of 5 arc minutes. In the present study, EGM2008 contribution is utilized up to degree and order of expansion 2159. According to recent studies, the maximum degree 2190 showed only minor improvements in the Hellenic area (Tziavos et al., 2010). Figure 10 presents the differences at the 103 benchmarks in Attica region. A mean value of -0.362 m is calculated. This bias represents the W_0 offset of the Greek vertical datum with respect to EGM2008. The internal accuracy of the procedure can be expressed by the standard deviation of the differences computed ±0.083 m in Attica region.

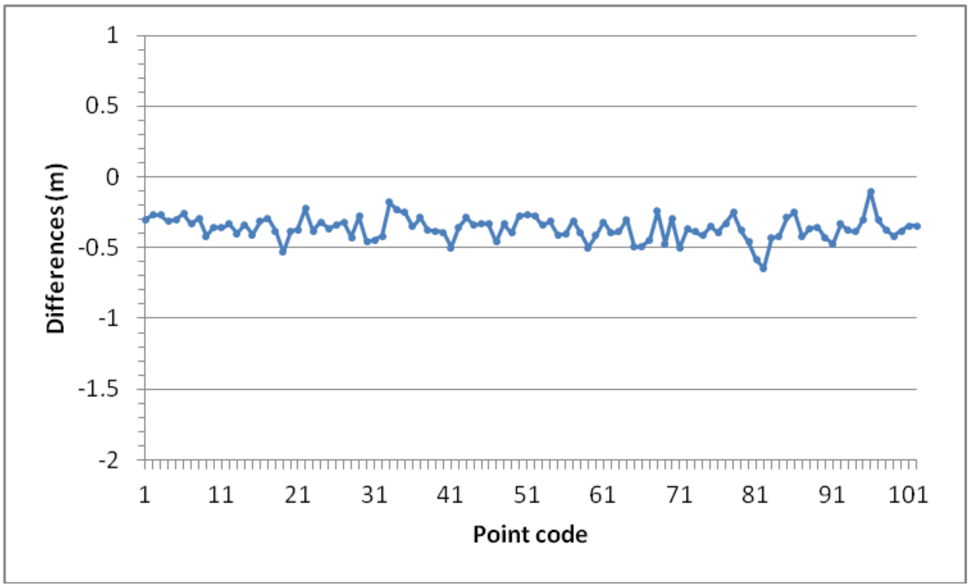

Figure 10: The differences between GPS/levelling and geopotential model derived geoid at Attica test area after blunders removal

Approximately the same situation is presented in the test area of Thessaloniki. The differences between GPS/leveling and the GGM geoid heights are charted in Figure 11. The statistics of the 127 point differences demonstrated a mean of -0.588 m and ±0.158 m standard deviation. The clarification of a bias difference (approximately 0.20 m) between Attica and Thessaloniki area results is part of our future research plan related to the unification of the Greek Local Vertical Datum (LVD). At first glance, it can be attributed to datum inconsistencies in the vertical datum. The standard deviation of the differences ±0.158 m reveals an accuracy degradation from the results of Attica which is attibuted to the fact that the study area of Thessaloniki has rougher terrain and it is characterized by higher elevations. Hence, orthometric heights are, naturally, of lower accuracy.

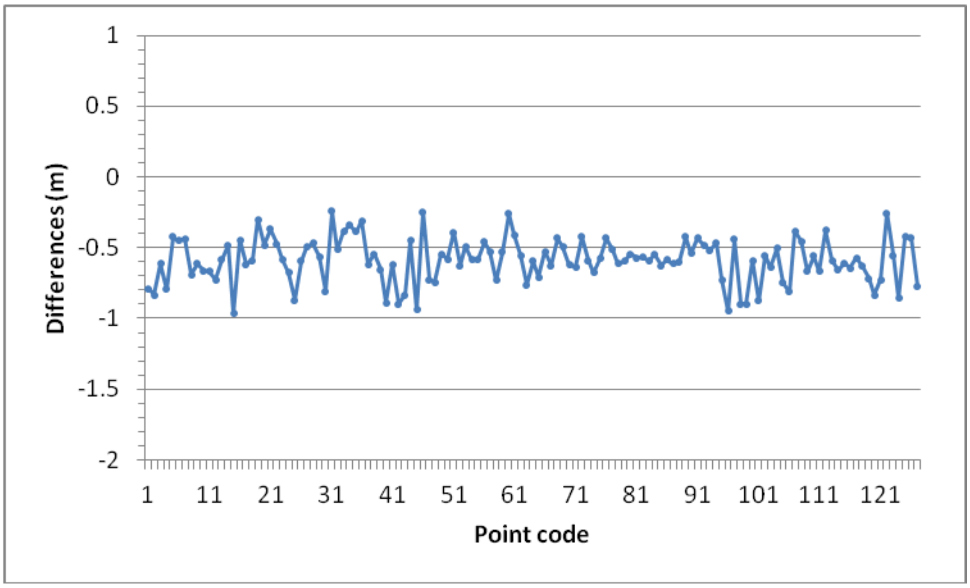

Figure 11: The differences between GPS/levelling and geopotential model derived geoid at Thessaloniki test area

5. Conclusions – Future Plans

The investigation of the internal as well as the external accuracy of the Hellenic vertical network is the main goal of E.LE.V.A.T.I.O.N. project. Two test areas are chosen and the initial assessment of the internal accuracy of the network is based on GPS measurements at benchmarks with known orthometric heights. Different GPS processing software packages are used and compared to each other. The global geopotential model EGM2008 is utilized for the assessment of the external accuracy of the network. Two test areas are chosen in Central and Northern Greece containing 230 benchmarks in total.

Based on the discussion of the GPS processing, some conclusions related to the performance of different software packages can be drawn. In the case of the baselines tested here (short baselines, a few tens of kilometers in length, observed for 1 hour) the commercial software packages perform better than the scientific one. The requirement of increased amount of data for the proper modeling of a large number of parameters estimated by the scientific software is the main reason for its reduced performance. Under unfavorable measurement conditions (reduced satellite visibility and/or poor signal reception) there are noticeable differences in the performance of the various software packages. Differences exist among the commercial software packages based on the solution strategy of each one of them, depending on the baseline length and the observation period. Some of these differences can be attributed to the processing engine, which is by design more suitable for kinematic measurements than for static ones.

The difficult measurement environment clearly affects the precision of the final result. This fact stands for all software packages used in our study. The precision degradation is found higher for the horizontal coordinates rather than for the heights, as the vertical component is always estimated with reduced accuracy. This fact underlines the importance of the observation conditions during a GPS campaign. A careful planning of the measurements is of great importance for high precision applications. Nevertheless, generally speaking, the horizontal closure errors are smaller than the vertical closures. However, certain software programs provided slightly better results in the vertical component. This remark requires further investigation.

The validation of the vertical datum in both test areas is performed using external information from the state-of-the-art global geopotential model EGM2008. The results in Attica show an agreement between "geometric" and "physical" geoid of 8.3.cm, in terms of the standard deviation of the differences. In Thessaloniki, this agreement is 15.8 cm. A bias between the average difference of Attica and Thessaloniki is observed, which can be attributed to the datum offset between the Greek datum and EGM2008. This bias presents different characteristics in Attica than in Thessaloniki, resulting a 20 cm offset, approximately, between the average differences at the two areas. The abovementioned offset is related to the LVD used in each area and it is the subject of ongoing work. It should be noted that due to the absence of sufficient documentation and the repeated partial adjustments performed since its creation, the actual accuracy of the Hellenic vertical datum is largely unknown. The use of additional geopotential models, especially the recent available models from GOCE satellite, will contribute to the efficient validation of the height datum with respect to its spectral characteristics.

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